Unit 6 Review – Relationships within Triangles

Perpendicular Bisector

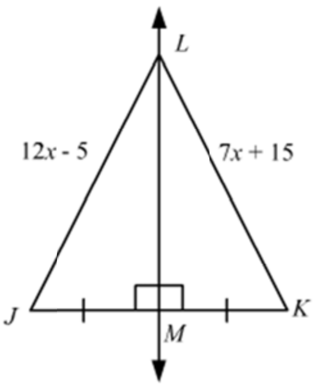
Definition: a line that goes through the midpoint of a line segment and is also perpendicular to that line segment.

Property: every point on a perpendicular bisector is equidistant to the endpoints of the segment it bisects.

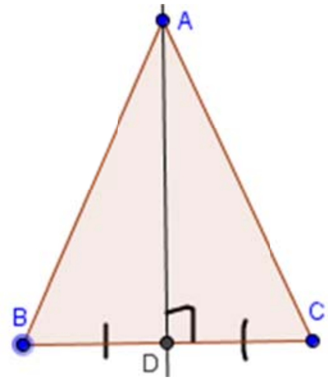
Point of concurrency: Circumcenter

Property of the Circumcenter – Equidistant to the vertices.

1. Solve for x.



2. Can you assume ABC is isosceles?



3. Write the equation of the perpendicular bisector through points:

a) (1, 5) (7, -1)

b) (-3, 4) (9, 8)

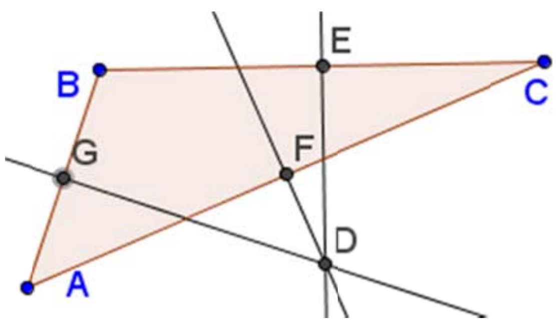
4. Find the coordinates of the circumcenter of the triangle with the given vertices.

a) A(-10, 7) B(-6, 3) C(-2, 3)

b) D(3, -6) E(5, -3) F(8, -6)

c) G(0, 0) H(8, 6) I(2, 2)

5.



5. Find x and y given segments GD, ED and FD are perpendicular bisectors:

a) FC = 2x + 7, AC = 4y + 6, BD = 3x + 1, DC = y + 5

b) DA = 13 – x, BD = y – 8, DC = 11

c) CD = x + y, BD = 11y - x, DA = 12

Angle Bisector

Definition: A line that divides an angle into two congruent angles.

Property: Every point on a perpendicular bisector is equidistant to the sides of the angle it bisects.

Point of Concurrency: Incenter

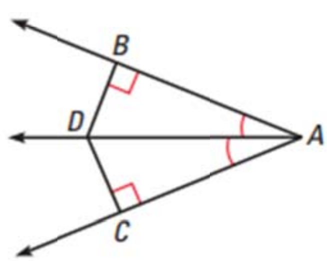
Property of the Incenter: Equidistant to the sides of the triangle.

6. **Challenge** Use the diagram and the information below to prove the Angle Bisector Theorem.

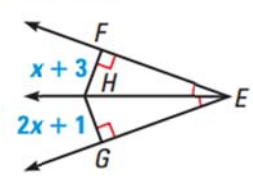
Given ▸ D is on the bisector of $\angle BAC$. $\overline{DB} \perp \overrightarrow{AB}$, $\overline{DC} \perp \overrightarrow{AC}$

Prove ▸ $\overline{DB} \cong \overline{DC}$

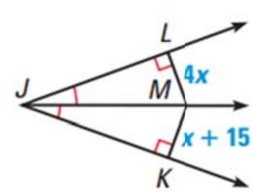
Hint: First prove that $\triangle ADB \cong \triangle ADC$.



7. Find *FH*.



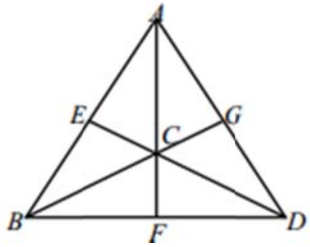
8. Find *MK*.



Median

Definition: A line connecting a vertex of a triangle to the midpoint of the opposite side.
Property: None
Point of Concurrency: Centroid
Property of a Centroid: Centroid is twice the distance from the vertex as it is from the opposite side.

9. If $BG = 24$, what is the length of $\overline{CG}$?
If $BC = 9$ and $CG = 2x + 1$, solve for x.
If $AC = y$ and $CF = 2.5$, solve for y.
If $CD = 14.4$ and $EC = 8z$, solve for z.



10. Given the vertices of the triangle, find the midpoint of each segment and the coordinates of the centroid.
a) A(0, 6) B(4, 10) C(2, 2)
b) D(1, 5) E(-2, 7) F(-6, 3)
c) X(1, 4) Y(7, 2) Z(2, 3)

Altitude

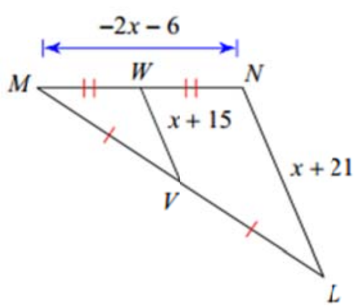
Definition: A line segment from the vertex of triangle perpendicular (or a line coincident) to the opposite side.
Property: None
Point of Concurrency: Orthocenter
Property of an Orthocenter: If the triangle is acute, the orthocenter is inside the triangle.
If the triangle is right, the orthocenter is on the triangle.
If the triangle is obtuse, the orthocenter is outside the triangle.

11. Find the coordinates of the orthocenter given the vertices of the triangle.
a) L(0, 5) M(3, 1) N(8, 1)
b) A(-4, 0) B(1, 0) C(-1, 3)
c) T(-2, 1) U(2, 1) V(0, 4)

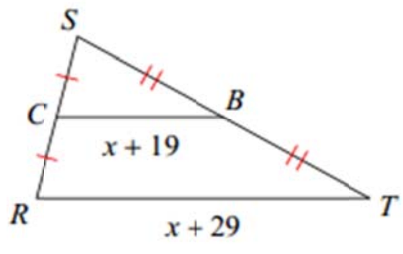
Midsegment

Definition: A line segment connecting two midpoints of two sides of a triangle.
Property: 1. The midsegment is parallel to the third side of the triangle.
2. The midsegment is half the length of the third side of the triangle.

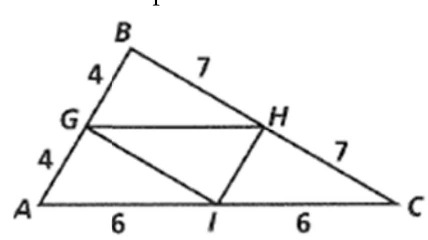
12. Find VW



13.



14. Find the perimeter of GHI



Also review section 6.5 Inequalities in Triangles and Indirect Proof by looking at the worksheet from last class. Main ideas from that section include the longer side theorem, larger angle theorem, triangle inequality conjecture, hinge theorem and indirect proofs.