

GEOMETRY REGULAR POLYGONS

A **polygon** is a closed plane figure with at least three sides. The sides intersect only at their endpoints and no adjacent sides are collinear.

A **regular polygon** is equilateral (all sides congruent) and equiangular (all angles congruent).

Directions: Complete the following table. Some hints are given below.

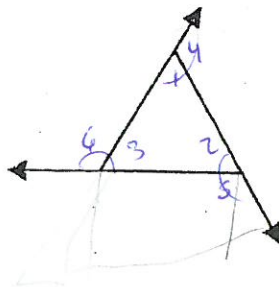
Polygon	1. Number of Sides	2. Number of Angles	3. Number of Triangles Formed	4. Sum of Interior Angles	5. ^A One Interior Angle of Regular Polygon	6. Sum of Exterior Angles	7. ^A One Exterior Angle of Regular Polygon
Triangle	3	3	1	180	60	360	120
Quadrilateral	4	4	2	360	90	360	90
Pentagon	5	5	3	540	108	360	72
Hexagon	6	6	4	720	120	360	60
Octagon	8	8	6	1080	135	360	45
Decagon	10	10	8	1440	144	360	36
Dodecagon	12	12	10	1800	150	360	30
n-gon	n	n	n-2	$(n-2)180$	$\frac{(n-2)180}{n}$	360°	$\frac{360^\circ}{n}$

Hints:

1. Complete columns 1 through 5 first.

- You already know the answers for a triangle.
- Sketch polygons with 4, 5, 6, 8, 10, and 12 sides.
- Divide each polygon into triangles by drawing all the diagonals from one vertex. Use this to help you determine the sum of the interior angles.

2. For columns 6 and 7, extend each side of the polygon so that there is one exterior angle at each vertex. For example, a triangle will look like the following.



Examples

1. Find the measures of each interior angle.

a)

$(6-2)180^\circ$
 $4(9x+30) + 2(2x) = 720^\circ$
 $36x + 120 + 4x = 720^\circ$
 $40x = 600$
 $\frac{40x}{40} = \frac{600}{40}$
 $x = 15^\circ$

b)

$6x = 360^\circ$
 $\frac{6x}{6} = \frac{360}{6}$
 $x = 60^\circ$

2. Find the measure of each interior angle and each exterior angle of a regular 15-gon.

$Sum = (15-2)180^\circ = 2340^\circ$
 each int L = $\frac{2340}{15} = 156^\circ$ each ext L = $24^\circ = \frac{360}{15}$

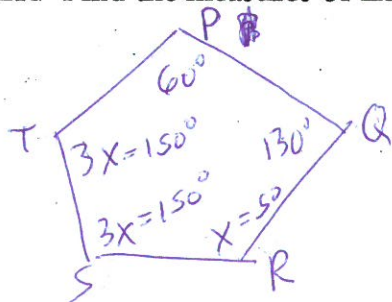
3. Six angles of a convex octagon are congruent. Each of the other two angles has a measure 20 more than the measure of each of the other six angles. Find the measure of each angle.

$6x^\circ + 2(x^\circ + 20) = (8-2)180$
 $6x + 2x + 40 = 6 \cdot 180$
 $8x + 40 = 1080$
 $-40 \quad -40$
 $8x = 1040$
 $\frac{8x}{8} = \frac{1040}{8}$
 $x = 130^\circ$
 150°

4. If the sum of the interior angles of a regular polygon is 1080, find (a) the number of sides, (b) one exterior angle, and (c) one interior angle.

$\frac{(n-2)180}{180} = \frac{1080}{180}$
 $n-2 = 6 \rightarrow n = 8$
 b) $\frac{360}{8} = 45^\circ$
 c) 135°

5. In Pentagon PQRST, $m\angle P = 60$ and $m\angle Q = 130$. $\angle S$ and $\angle T$ are each three times as large as $\angle R$. Find the measures of $\angle R$, $\angle S$ and $\angle T$.



$2 \cdot 190^\circ + 7x = (5-2)180$
 $3 \cdot (180)$
 $190 + 7x = 540$
 $-190 \quad -190$
 $7x = 350$
 $\frac{7x}{7} = \frac{350}{7}$
 $x = 50^\circ$