Name _____ Precalc Review of Parametric Equations

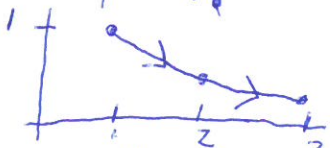
1. Graph BY HAND the following sets of parametric equations using your calculator to check.

a. $\begin{cases} x = 4 - 2t \\ y = 3 + 6t - 4t^2 \end{cases} \quad -4 \le t \le 4$

t	-4	-3	-2	-1	0	1	2	3	4
x	12	10	8	6	4	2	0	-2	-4
y	-85	-51	-25	-7	3	5	-1	-15	-37

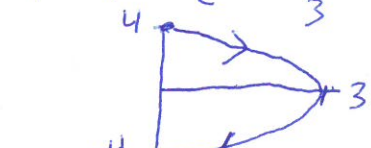
b. $\begin{cases} x = \sqrt{t+1} \\ y = \dfrac{1}{t+1} \end{cases} \quad 0 \le t \le 8$

t	0	1	2	3	4	5	6	7	8
x	1		2						3
y	1		1/4						1/9



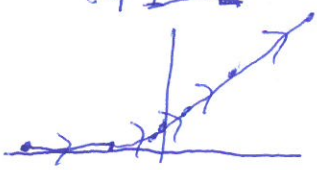
c. $\begin{cases} x = 3\sin(\frac{t}{2}) \\ y = 4\cos(\frac{t}{2}) \end{cases} \quad 0 \le t \le 2\pi$

t	0	π/2	π	3π/2	2π
x	0	$3\sqrt{2}/2$	3	$3\sqrt{2}/2$	0
y	4	$4\sqrt{2}/2$	0	$-4\sqrt{2}/2$	-4



d. $\begin{cases} x = t^3 \\ y = 2^t \end{cases} \quad t \in [-3, 3]$

t	-3	-2	-1	0	1	2	3
x	-27	-8	-1	0	1	8	27
y	1/8	1/4	1/2	1	2	4	8



2. a. Find a rectangular equation by eliminating the parameter for all of the equations in #1.
b. Sketch the graphs of the rectangular equations. How do the graphs differ from those in #1. → No arrows / continuous domain

a) $-4\left(\frac{x-4}{-2}\right)^2 + 6\left(\frac{x-4}{-2}\right) + 3 = y$ b) $y = \frac{1}{x^2}$ c) $\frac{x^2}{9} + \frac{y^2}{16} = 1$

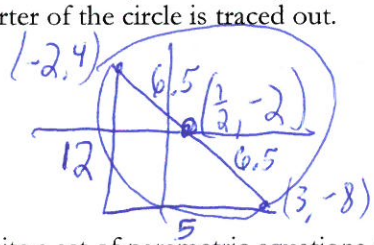
3. Find a set of parametric equations for the given rectangular equations. Let t = x + 2 be your parameter.

a. $y = 3x - 2$ $\begin{cases} x = t - 2 \\ y = 3t - 8 \end{cases}$

b. $y = x^2$ $\begin{cases} x = t - 2 \\ y = t^2 - 4t + 4 \end{cases}$ $x = t - 2$

c. $x = y^{\frac{5}{4}}$ $x = t - 2$, $y = (t-2)^{4/5}$

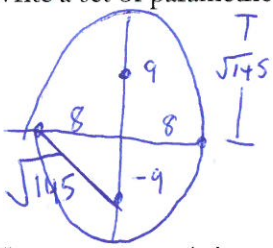
4. Write a set of parametric equations for a circle which has a diameter with endpoints (-2, 4) and (3, -8) and a domain such that only a quarter of the circle is traced out.



$x = \frac{13}{2}\cos(\theta) + \frac{1}{2}$

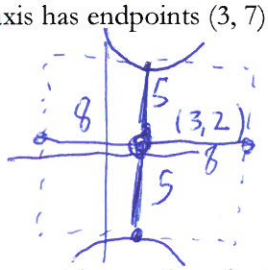
$y = \frac{13}{2}\sin(\theta) - 2$

5. Write a set of parametric equations for an ellipse which has a minor axis length 16 and foci at (0,9) and (0, -9).



$\frac{x^2}{64} + \frac{y^2}{145} = 1$

6. The transverse axis has endpoints (3, 7) and (3, -3) and the conjugate axis has length 16. Write a set of parametric equations for this conic section.



$\frac{(y-3)^2}{25} - \frac{(x-2)^2}{64} = 1$

7. Write a set of parametric equations for the line through the points (-2, 7) and (3, 22) given the parameter t = 2x – 1

$m = \frac{22-7}{3-(-2)} = \frac{15}{5} = 3$

$y = 3x + 13$

$x = \frac{t+1}{2}$

$y = 3x + b$

$7 = (3)-2 + b$

$+6 \quad +6$

$b = 13$

$y = 3\left(\frac{t+1}{2}\right) + 13$

8. Write a set of parametric equations for each of the following.

a. $\frac{(x+3)^2}{64}+\frac{(y-2)^2}{20}=1$

$x = 8\cos\theta - 3$

$y = 2\sqrt{5}\sin\theta + 2$

b. $(x-1)^2+(y)^2=36$

$x = 6\cos\theta + 1$

$y = 6\sin\theta$

$\sec^2\theta - \tan^2\theta = 1$

c. $\frac{(x+3)^2}{16}-\frac{(y-2)^2}{25}=1$

$x = 4\sec\theta - 3$

$y = 5\tan\theta + 2$

d. $\frac{(y+2)^2}{100}-\frac{(x-7)^2}{121}=1$

$x = 11\tan\theta + 7$

$y = 10\sec\theta - 2$

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

9. Eliminate the parameter in each of the following.

a. $\begin{cases} x = \sqrt{29}\cos(t) \\ y = 6\sin(t) - 2 \end{cases}$

$\frac{x^2}{29} + \frac{(y+2)^2}{36} = 1$

b. $\begin{cases} x = 5\tan(t) - 3 \\ y = 8\sec(t) - 1 \end{cases}$

$\frac{(y+1)^2}{64} - \frac{(x+3)^2}{25} = 1$

c. $\begin{cases} x = 5\csc(t) + 2 \\ y = \cot(t) - 3 \end{cases}$

$\csc^2\theta - \cot^2\theta = 1$

$\frac{(x-2)^2}{25} - \frac{(y+3)^2}{1} = 1$

10. A person goes up an escalator with a horizontal speed of 1 ft/s and a vertical speed of 2 ft/s.

a. Find a set of parametric equations for the motion of the helicopter.

$x = t$

$y = 2t$

b. Describe the location of the person at t = 7 seconds. (7, 14)

11. From his starting point, a biker rides along a straight path. His speed to the north is 2 mi/h. Her speed to the east is 1.4 mi/h. Let x represent how far east of her starting point the hiker is, and let y represent how far north she is.

a. Find a set of parametric equations for his motion. $\begin{cases} x = 2t \\ y = 1.4t \end{cases}$

b. Write an equation in x and y only (rectangular) for his motion. $y = \frac{1.4}{2}x$

c. Find the location of the biker 90 minutes into his trip. (3, 2.1)

1.5 hours