Name____ Period:______

Parametric Equations Notes

Draw a graph to represent each set of parametric equations. Be sure to show the direction. $-2 \le t \le 2$

1. $\begin{cases} x = 4t \\ y = 2t \end{cases}$ 2. $\begin{cases} x = t - 2 \\ y = 4t \end{cases}$ 3. $\begin{cases} x = \dfrac{t}{4} \\ y = -3t \end{cases}$ 4. $\begin{cases} x = 20t \\ y = 10t + 10 \end{cases}$

1.

t	-2	-1	0	1	2
X					
Y					

2.

t	-2	-1	0	1	2
X					
Y					

3.

t	-2	-1	0	1	2
X					
Y					

4.

t	-2	-1	0	1	2
X					
Y					

5. Suppose a research submarine descends from the surface with a horizontal speed of 1.8 m/s and a vertical speed of 0.9 m/s.

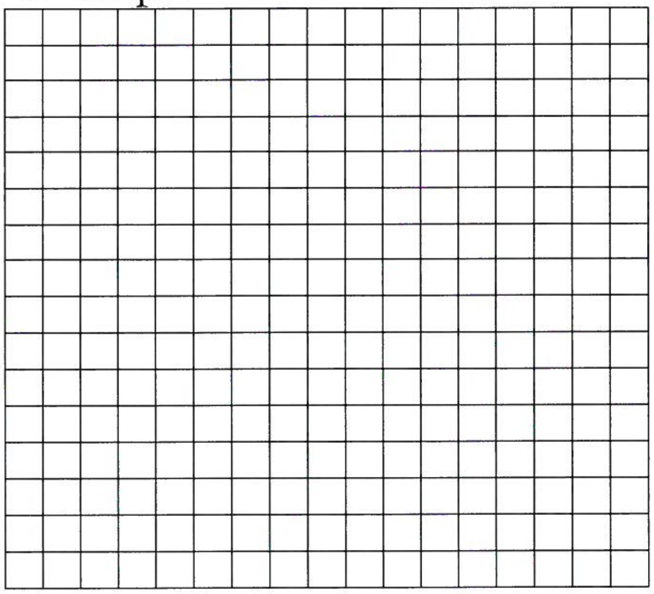
a. Write equations for and draw a graph of the motion of the submarine.

b. Find the depth of the submarine after 50s.

c. Find the submarine's depth after 1 day. Does this answer make sense? Explain.

6. Graph $x = 2t$ and $y = t^2 + 1$, on $[-2, 5]$

7. Graph $x = t^2$ and $y = t - 1$ for $-3 \leq t \leq 3$.



8. Eliminating the Parameter.

Sometimes we need to look at a single two-variable equation of a curve instead of parametric equations. To do this we eliminate the parameter by solving one equation for t and substituting into the other equation. Eliminate the parameter for the equations in problems 1-7.

9. Finding Parametric Equations for a graph.
We now go in the other direction. We are given an equation in x and y and we find parametric equations for the same curve. There are an infinite amount of these equations that can be made without a given parameter.
Example:

Find a set of parametric equations that represent $y = x^2 + 1$ by using the following parameters.

a. $t = x$

b. $t = x - 2$

10. Given the points (-1,3) and (2,5). Find the equation of the line passing through the two points. Then, find a set of parametric equations for the line for the following given parameters. A) $x = t - 3$ B) $t = 2x$

Parametric Equations and Conic Sections

Parabola:

Easiest type, just choose the parameter you want to use and set x equal to it. Then y must be equal to the square of that. Here are some examples that you should graph with your calculator:

$$\begin{cases} x_1(t) = t \\ y_1(t) = t^2 \end{cases} \qquad \begin{cases} x_2(t) = \cos(t) \\ y_2(t) = \cos^2(t) \end{cases} \qquad \begin{cases} x_3(t) = e^t \\ y_3(t) = e^{2t} \end{cases}$$

Notice that each of these parabolas will be different. That's the thing with parametrics. Unless you make the most mundane selection of the parameter you get interesting little sections of the graph, and of course, the direction in which the graph is traced out will change. For (x_1, y_1) we get the parabola we're used to; (x_2, y_2) will trace out a parabola whose domain is $-1 \le x \le 1$ because of the *range* of cosine; (x_3, y_3) is only the right-hand side of the parabola because of the *range* of e^t. Also, while the first and third graphs are traced out in only one direction (from left to right), the second graph just keeps going back and forth on itself.

Circle:

Second easiest. Think about the unit circle that you spent hours and hours working with. For that we had

$$\cos(t) = \frac{x}{r} \qquad \sin(t) = \frac{y}{r}$$

Now we'll just rearrange and get parametric equations for circles:

$$\begin{cases} x = r\cos(\theta) + h \\ y = r\sin(\theta) + k \end{cases}$$

We can change the radius of the circle by changing r.

11. Write a set of parametric equations for a circle of radius 12 restricting the domain so that only a quarter of the circle is traced out. Repeat for 3/4 of the circle.

12. Write parametric equations for a circle with equation $(x-2)^2 + (y+4)^2 = 9$.

13. Write parametric equations for a circle that is centered at $(2, 5)$ and has a radius of 4.

Ellipse:
An ellipse is a lot like a circle, just that the radius doesn't stay fixed. In fact, the parametric equations for an ellipse are almost identical to the parametric equations for a circle. Instead of using r for a fixed radius, we use *a* and *b*. Here's an ellipse:

$$\begin{cases} x = a\cos(t) + h \\ y = b\sin(t) + k \end{cases}$$

14. Write a set of parametric equations for the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

15. Write a set of parametric equations for the ellipse $\frac{(x-3)^2}{4} + \frac{(y+1)^2}{64} = 1$.

Hyperbola:
Our choices for parameterization of a hyperbola are slightly more involved than for an ellipse or circle. We base our choices on the other Pythagorean Identity: $\sec^2(t) - \tan^2(t) = 1$. Our selection of parameterization will determine whether the hyperbola is vertical or horizontal.
Type I: Horizontal

$$\begin{cases} x = a\sec(t) \\ y = b\tan(t) \end{cases}$$

We can eliminate the parameter to see why this is a horizontal hyperbola:

$$\left(\frac{x}{a}\right)^2 = \sec^2(t) \qquad \left(\frac{y}{b}\right)^2 = \tan^2(t)$$

From these we can perform the appropriate subtraction and end up with

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2(t) - \tan^2(t)$$

so we get,

$$\begin{cases} x = a\sec(t) \\ y = b\tan(t) \end{cases} \Leftrightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Type II: Vertical

$$\begin{cases} x = a\tan(t) \\ y = b\sec(t) \end{cases}$$

We can go through the same type of procedure to eliminate the parameter again and end up with:

$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, which is a vertical hyperbola.

Summary:

The parametric equations for conic sections centered at (h,k) are given by:

Circle

$$\begin{cases} x = r\cos(t) + h \\ y = r\sin(t) + k \end{cases}$$

Ellipse

$$\begin{cases} x = a\cos(t) + h \\ y = b\sin(t) + k \end{cases}$$

Horizontal Hyperbola

$$\begin{cases} x = a\sec(t) + h \\ y = b\tan(t) + k \end{cases}$$

Vertical Hyperbola

$$\begin{cases} x = a\tan(t) + h \\ y = b\sec(t) + k \end{cases}$$

Find parametric equations for each of the following.

16. $(x+3)^2 + (y-1)^2 = 16$

17. $\frac{(x+5)^2}{8} + \frac{y^2}{12} = 1$

18. $\frac{x^2}{20} - \frac{(y+2)^2}{12} = 1$

19. $\frac{(x-5)^2}{16} + \frac{(y+2)^2}{25} = 1$

20. $\frac{(y-8)^2}{36} - \frac{(x+1)^2}{49} = 1$

21. $x^2 + y^2 - 6x - 2y - 10 = 0$

Eliminate the parameter in each of the following.

22. $\begin{cases} x = 3\cos(t) + 2 \\ y = 5\sin(t) - 3 \end{cases}$

23. $\begin{cases} x = 8\sin(t) - 4 \\ y = 5\cos(t) + 8 \end{cases}$

24. $\begin{cases} x = 3\tan(2t) \\ y = 5\sec(2t) \end{cases}$

25. $\begin{cases} x = 3\sec(t) + 9 \\ y = 5\tan(t) - 8 \end{cases}$

26. $\begin{cases} x = 12\cos(5t) + 4 \\ y = 12\sin(5t) - 1 \end{cases}$

27. $\begin{cases} x = 3\csc(t) + 2 \\ y = 8\cot(t) - 3 \end{cases}$

28. $\begin{cases} x = 4 - 3\cot(t) \\ y = 5\csc(t) - 4 \end{cases}$

29. $\begin{cases} x = 5\cos(t) + 4 \\ y = 4 - 5\sin(-t) \end{cases}$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

30. Graph the following on your calculator.

$x = 16 \sin^3 t$
$y = 13 \cos t - 5 \cos(2t) - 2\cos(3t) - \cos(4t)$

31. Eliminate the parameter t from $x = 3t$ and $y = t^2$.

32. Eliminate the parameter from the parametric equations $x = \sqrt{t+1}$, $y = 3t + 2$.

33. Graph the parametric equations $x = 1 + \sqrt{t}$, $y = t^2 - 4t$ on $0 \leq t \leq 4$.

34. Which of the following are the parametric equations to describe an object travelling along the line passing through (1,0) and (2,3)?
There may be more than one correct answer.

a) ☐ $x = 2 + t$, $y = 3 + 3t$ b) ☐ $x = 2 + 3t$, $y = 3 + 3t$

c) ☐ $x = 1 + t$, $y = 3t$ d) ☐ $1 + 3t$, $y = t$

35.

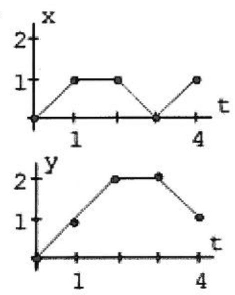


Fig. 23

For problems 5–8, use the data in the given graphs of (t, x(t)) and (t, y(t)) to sketch the parametric graph (x(t), y(t)).

5. Use x and y from Fig. 23.

6. Use x and y from Fig. 24.

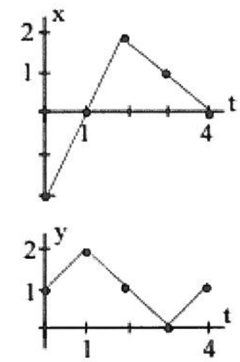


Fig. 24

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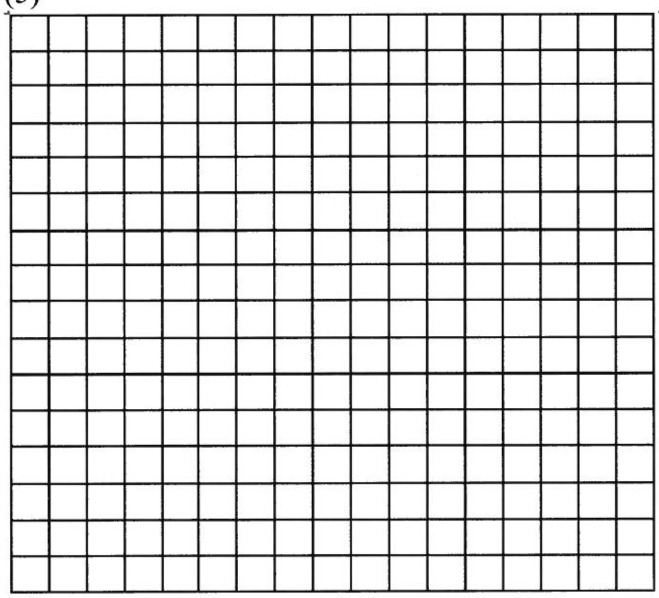

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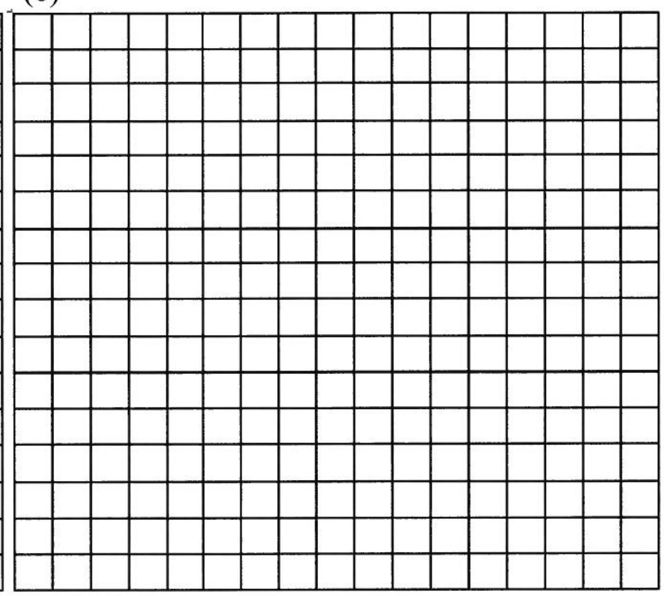

36. Convert the following to a rectangular equation.

$$x = -t + 1$$
$$y = -4t^2 - 3$$

37. Graph the following sets of parametric equations on a piece of graph paper using your calculator to check. Make a table of values with $-3 \le t \le 3$.

$a. \begin{cases} x = t \\ y = t + 2 \end{cases}$ $\quad b. \begin{cases} x = t \\ y = -\dfrac{3}{4}t \end{cases}$ $\quad c. \begin{cases} x = t^2 \\ y = 2 - t \end{cases}$ $\quad d. \begin{cases} x = \sqrt{t} \\ y = 2 - t \end{cases}$

38. Convert each of the parametric equations in 37 to rectangular equations.

39. Find two different sets of parametric equations for the given rectangular equations.

a. $y = 3x - 2$ *b.* $y = x^2$ *c.* $x = y^{\frac{5}{4}}$

40. Write a set of parametric equations for a circle of radius 8 restricting the domain so that only half of the circle is traced out.

41. Write a set of parametric equations for each of the following.

a. $\dfrac{(x-2)^2}{49} + \dfrac{(y+6)^2}{25} = 1$ b. $(x+3)^2 + (y-2)^2 = 16$

c. $\dfrac{(x-5)^2}{4} - \dfrac{(y+4)^2}{36} = 1$ d. $\dfrac{(y+9)^2}{8} - \dfrac{(x-7)^2}{81} = 1$

42. Eliminate the parameter in each of the following.

a. $\begin{cases} x = 2\cos(t) + 3 \\ y = 6\sin(t) - 2 \end{cases}$ *b.* $\begin{cases} x = 6\tan(t) + 2 \\ y = 3\sec(t) - 7 \end{cases}$ *c.* $\begin{cases} x = 3\csc(t) + 2 \\ y = 8\cot(t) - 3 \end{cases}$

43. A helicopter takes off with a horizontal speed of 5 ft/s and a vertical speed of 20 ft/s.

a. Find a set of parametric equations for the motion of the helicopter.

b. Describe the location of the helicopter at t = 10 seconds. ______________________________

44. From her starting point, a hiker walks along a straight path. Her speed to the north is 3 mi/h. Her speed to the east is 0.4 mi/h. Let x represent how far east of her starting point the hiker is, and let y represent how far north she is.

a. Find a set of parametric equations for her motion. $\begin{cases} x = \\ y = \end{cases}$

b. Write an equation in x and y only (rectangular) for her motion. __________________________

c. Find the location of the hiker 90 minutes into her trip. __________________________