Chapter 6 Cumulative Review: Solving Trigonometric Equations

General Strategy

1) Is there only one trigonometric function in the equation you are trying to solve? If yes, solve it algebraically using reverse PEMDAS if the equation is linear or factoring/quadratic formula if the equation is quadratic. If no, proceed to step 2.
2) Is there one trigonometric function present in each term (terms are expressions being added)? If yes, factor it out using the reverse of the distributive property. Then set each factor equal to zero and solve. You can get anywhere from 0 to 4 solutions (assuming the equation is either of degree 1 or 2). If no, proceed to step 3.
3) You now need to use identities to get the equation in the form of either step 1 or 2. But, sometimes that won't work. A prime example of this is when you see two different linear trigonometric functions in the equation. For example, $\sin x - \cos x = 1$. If this is the case, proceed to step 4.
4) Square both sides and go back to step 3.

See, it's simple!

Chapter 3 Review

When you get down to a trigonometric function equals a ratio, you need to know how to find out what angle(s) will satisfy your equation. It is something we have been doing all year and it involves 1) the reference angle and 2) the quadrant.

Ex. $\tan x = -\sqrt{3}$

1) First identify the reference angle x'. If the ratio is one from your chart, then that is how you find the reference angle. In this case x' = 60 degrees. If the ratio is not one from the chart, you must use your calculator. Remember to take the absolute value of the ratio when using the inverse function so that you get the angle in quadrant 1, the reference angle. Also, if the ratio is outside the range of the trigonometric function in question, then there is no solution.
2) Identify the quadrant. Use ASTC and you will have two non coterminal angles except when the reference angle gives you a ratio of 1 when you plug it back into your trigonometric function. The reason why sometimes you get two solutions vs one solution is because you can only construct two angles with 0 or 90 degree reference angles where you can construct four for any reference angle in between 0 and 90 degrees.

Now is every different scenario I can possibly think of when solving trigonometric equations. Find solutions for

1) $0^\circ \le \theta < 360^\circ$
2) $0 \le \theta < 2\pi$
3) For all θ.

Linear – Isolate the trigonometric function.

$\sqrt{3}\cot\theta - 1 = 0$

$4\cos\theta - 1 = 3\cos\theta + 4$

Quadratic – Factor or use the quadratic formula.

$2\cos^2\theta - \cos\theta + 1$

$6\sin^2\theta = \sin\theta + 1$

Identities – Get in terms of sine and cosine, then multiply each term by the common denominator.

$4\sin\theta = 2\csc\theta$

$2\cos\theta + \tan\theta = \sec\theta$

Double Angle Formulas – Use DA formulas when you see 2θ and θ in the same equation.

$\cos 2\theta - 3\sin\theta = 2$

$\sin 2\theta - \cos\theta = 0$

Pythagorean Identities – Use Pythagorean identities when you see two different trigonometric functions and only one is squared.

$2\cos^2\theta + \sin x - 1 = 0$

$4\sin^2\theta + 4\cos\theta - 5 = 0$

Multiple Angles – Solve using a previously stated strategy if all the angles are in the same form $x\theta$, then find all solutions.

$\sin 3\theta = -1$

$2\cos^2 2\theta - \cos\theta - 1 = 0$

Sum and Difference Formulas – If the equation looks like it is in the form of a sum or difference formula, then use the correct formula to condense one side of the equation to one trigonometric function.

$\sin 2\theta \cos\theta + \cos 2\theta \sin\theta = \frac{1}{2}$

$\cos 2\theta \cos\theta - \sin 2\theta \sin\theta = \frac{1}{\sqrt{2}}$

Square both sides – This will help create trigonometric functions squared so you can use identities. Don't forget to expand your binomials and not just square each term.

$\cos\theta - \sin\theta = 1$

$\sin\theta + \cos\theta = -1$