Chapter 5 Review

5.1 – Identities: You need to know the 8 fundamental identities and how to apply the 10 commandments (of identities.

5.2, 5.3, 5.4 – Sum, Difference, Double Angle and Half Angle Formulas.

Chapter 5 Trigonometry Formulas

$\sin(A+B)=\sin A\cos B+\cos A\sin B$

$\cos(A+B)=\cos A\cos B-\sin A\sin B$

$\cos(\frac{A}{2})=\pm\sqrt{\frac{1+\cos A}{2}}$

$\sin(A-B)=\sin A\cos B-\cos A\sin B$

$\cos(A-B)=\cos A\cos B+\sin A\sin B$

$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$

$\sin(2A)=2\sin A\cos A$

$\cos(2A)=\cos^2 A-\sin^2 A$

$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$

$\sin(\frac{A}{2})=\pm\sqrt{\frac{1-\cos A}{2}}$

$\cos(2A)=2\cos^2 A-1$

$\tan(2A)=\frac{2\tan A}{1-\tan^2 A}$

$\cos(2A)=1-2\sin^2 A$

For 1 – 4, prove the identity.

1) $\frac{\cos^4 x-\sin^4 x}{\sin^2 x}=\cot^2 x-1$

$\frac{(\cos^2 x-\sin^2 x)(\cos^2 x+\sin^2 x)}{\sin^2 x}=\cot^2 x-1$

$\frac{\cos^2 x}{\sin^2 x}-\frac{\sin^2 x}{\sin^2 x}$

$\cot^2 x-1$ ✓

2) $\csc B-\sin B=\cot B\cos B$

$\frac{1}{\sin B}-\frac{\sin B}{B}$

$\frac{1-\sin^2 B}{\sin B}$

$\frac{\cos^2 B}{\sin B}$

$\cos B\left(\frac{\cos B}{\sin B}\right)=\cot B\cos B$ ✓

3) $\cos(\frac{\pi}{2}+x)=-\sin x$

$\cos\frac{\pi}{2}\cos x-\sin\frac{\pi}{2}\sin x=-\sin x$

$0(\cos x)-1\sin x=-\sin x$

$-\sin x=-\sin x.$ ✓

For 4 – 7, Evaluate

4) $\tan(165) = \tan(120+45)$

$$\frac{\tan 120+\tan 45}{1-\tan 120\tan 45}$$

$$\frac{-\sqrt{3}+1}{1+\sqrt{3}} = \frac{1-\sqrt{3}}{(1+\sqrt{3})} \cdot \frac{(1-\sqrt{3})}{(1-\sqrt{3})}$$

$$\frac{1-2\sqrt{3}+3}{1-3} = \frac{4-2\sqrt{3}}{-2} = \frac{-2+\sqrt{3}}{\sqrt{3}-2}$$

5) $\csc(15)$

$\sin(45-30) = \sin 45\cos 30 - \cos 45\sin 30$

$\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)$

$\sin(15)$ $\frac{\sqrt{6}-\sqrt{2}}{4}$

$\csc 15 = \frac{4}{\sqrt{6}-\sqrt{2}} \cdot \frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}+\sqrt{2})} = \frac{4(\sqrt{6}+\sqrt{2})}{6-2} = \frac{4(\sqrt{6}+\sqrt{2})}{4}$

$\sqrt{6}+\sqrt{2}$

6) $\cos(\frac{\pi}{15})\cos(\frac{\pi}{10}) - \sin(\frac{\pi}{10})\sin(\frac{\pi}{15})$

$\cos\left(\frac{\pi}{15}+\frac{\pi}{10}\right) = \cos(12+18)$

$= \cos 30$

$= \frac{\sqrt{3}}{2}$

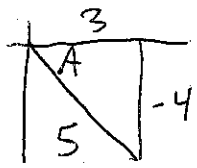
7) $\cos^2 195 - \sin^2 195$

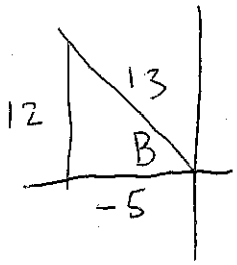
$\cos(2(195))$

$\cos 390$

$= \frac{\sqrt{3}}{2}$

8) $\sin A = -\frac{4}{5}$ with A in QIV and $\tan B = -\frac{12}{5}$ with B in QII, find:





a. $\cos(A-B)$

$\cos A\cos B + \sin A\sin B$

$\left(\frac{3}{5}\right)\left(\frac{-5}{13}\right) + \left(\frac{-4}{5}\right)\left(\frac{12}{13}\right)$

$\frac{-15-48}{65} = \frac{-63}{65}$

b. $\sin(2A)$

$= 2\sin A\cos A$

$2\left(\frac{-4}{5}\right)\left(\frac{3}{5}\right)$

$\frac{-24}{25}$

c. $\cot(A-B)$

$\tan(A-B) = \frac{\tan A-\tan B}{1+\tan A\tan B} = \frac{\left(\frac{-4}{3}\right)-\left(\frac{12}{-5}\right)}{1+\left(\frac{-4}{3}\right)\left(\frac{12}{-5}\right)}$

$= \frac{\frac{-20}{15}+\frac{36}{15}}{1-\frac{48}{15}} = \frac{\frac{16}{15}}{\frac{-33}{15}} = \frac{16}{-33}$

$\cot(A-B) = \frac{-33}{16}$

d. $\tan(2B)$

$= \frac{2\tan B}{1-\tan^2 B} = \frac{2\left(\frac{-12}{5}\right)}{1-\left(\frac{12}{5}\right)^2} = \frac{\frac{-24}{5}}{1-\frac{144}{25}} = \frac{\frac{-24}{5}}{\frac{-119}{25}}\ (25)$

$= \frac{-120}{-119} = \frac{120}{119}$

9) Graph $y = 2\sin 4x\cos 2x - 2\cos 4x\sin 2x$

$y = 2\sin(4x-2x)+1$

$y = 2\sin 2x + 1$

Period $= \frac{2\pi}{2} = \pi$