

# Trigonometric Ratios

In any triangle  $ABC$  where angle  $C$  measures  $90^\circ$ , we can define

$$\sin A = \frac{\text{side opposite } A}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{side adjacent to } A}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{side opposite } A}{\text{side adjacent to } A}$$

1. For each diagram, name (i) the hypotenuse, (ii) the side opposite  $\theta$ , (iii) the side adjacent to  $\theta$

a)  $H = AC$   
 $O = AB$   
 $A = CB$

b)  $H = ZX$   
 $O = YZ$   
 $A = YX$

c)  $H = QS$   
 $O = QR$   
 $A = RS$

2. In  $\triangle TRA$ ,  $\angle R$  is a right angle.

a) In terms of  $TR$ ,  $RA$ , and  $TA$ , find  $\sin T$ ,  $\cos T$ ,  $\tan T$ .  $\sin T = \frac{RA}{TA}$

b) In terms of  $TR$ ,  $RA$ , and  $TA$ , find  $\sin A$ ,  $\cos A$ ,  $\tan A$ .  $\cos T = \frac{TR}{TA}$   
 $\tan T = \frac{RA}{TR}$

3. Find  $\sin A$ ,  $\cos A$ , and  $\tan A$  in terms of  $p$ ,  $q$ , and  $m$  for each figure.

a)  $\sin A = \frac{p}{q}$   
 $\cos A = \frac{m}{q}$   
 $\tan A = \frac{p}{m}$

b)  $\sin A = \frac{p}{m}$   
 $\cos A = \frac{q}{m}$   
 $\tan A = \frac{p}{q}$

c)  $\sin A = \frac{m}{p}$   
 $\cos A = \frac{q}{p}$   
 $\tan A = \frac{m}{q}$

Make sure you are in degrees

4. Find the following trigonometric ratios. Give your answers correct to the ten-thousandth place.

- a)  $\sin(20^\circ) = .3420$     b)  $\sin(25.3^\circ) = .4274$     c)  $\sin(42.7^\circ) = .6782$
- d)  $\cos(20^\circ) = .9397$     e)  $\cos(20.4^\circ) = .9373$     f)  $\cos(28.4^\circ) = .8796$
- g)  $\tan(45^\circ) = 1$     h)  $\tan(64.4^\circ) = 2.0872$     i)  $\tan(50.5^\circ) = 1.2131$
- j)  $\sin(46^\circ) = .7193$     k)  $\tan(35^\circ) = .7002$     l)  $\cos(28^\circ) = .8829$
- m)  $\sin(24.5^\circ) = .4147$     n)  $\tan(43.4^\circ) = .9457$     o)  $\cos(54.8^\circ) = .5764$

5. Find  $\angle A$  to the nearest tenth of a degree if:

a)  $\sin A = 0.3542$

b)  $\cos A = 0.3542$

c)  $\tan A = 1.2450$

d)  $\sin A = 0.8702$

e)  $\cos A = 0.8702$

f)  $\tan A = 10.4569$

g)  $\sin A = 0.5000$

h)  $\cos A = 0.5000$

i)  $\tan A = 25.5000$

j)  $\sin A = 0.1452$

k)  $\cos A = 0.1452$

l)  $\tan A = 63.9800$

m)  $\sin A = 0.7249$

n)  $\cos A = 0.7249$

o)  $\tan A = 1543.2555$

46.5

43.5

~~89.1~~ 90.0

6. Find the measure of  $\angle A$  to the nearest tenth of a degree if:

a)  $\sin A = \cos(25^\circ) \cdot \tan(10^\circ) = .598$

b)  $\cos A = \tan(45^\circ) \cdot \tan(30^\circ) = .5774$

c)  $\cos A = \cos(53^\circ) \cdot \tan(46^\circ) = .623$

d)  $\sin A = \sin(30^\circ) \cdot \tan(30^\circ) = .289$

e)  $\tan A = \cos(63^\circ) \cdot \tan(82^\circ) = 3.23$

f)  $\tan A = \tan(17^\circ) \cdot \tan(73^\circ) = 1$

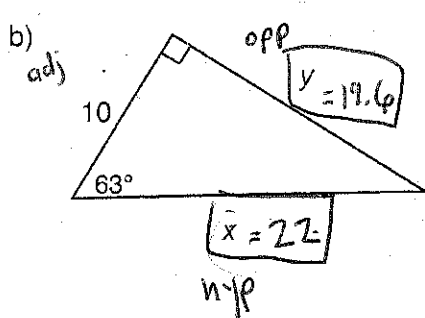
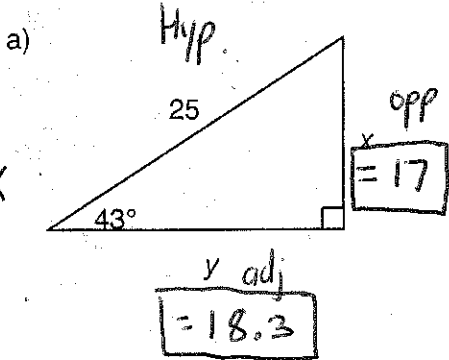
g)  $\tan A = \tan(25^\circ) \cdot \tan(65^\circ) = 1$

h)  $\tan A = \tan(70^\circ) \cdot \cos(45^\circ) = 1.94$

7. In the given right triangles, find the values of  $x$  and  $y$ . Give your answers correct to the hundredths place.

$\sin 43 = \frac{x}{25}$   
 $(.682)25 = x$

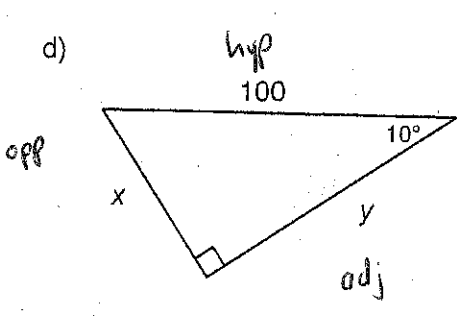
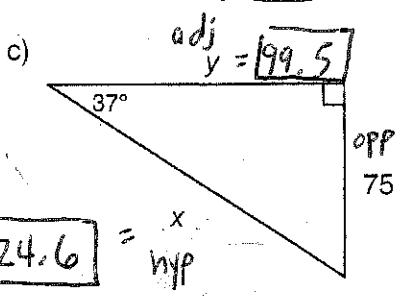
$\cos 43 = \frac{y}{25}$   
 $(.7314)25 = y$



$\tan(63) = \frac{y}{10}$   
 $y = \tan(63) \cdot 10 = 19.6$   
 $\cos(63) = \frac{10}{x}$   
 $x = \frac{10}{\cos(63)} = 22$

$\sin(37) = \frac{75}{x}$   
 $x = \frac{75}{\sin(37)} = 124.6$

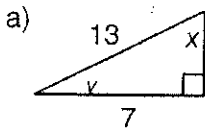
$\tan(37) = \frac{75}{y}$   
 $y = \frac{75}{\tan(37)} = 99.5$



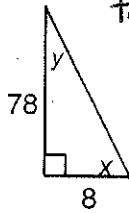
$\sin(10) = \frac{x}{100}$   
 $x = \sin(10) \cdot 100 = 17.4$   
 $\cos(10) = \frac{y}{100}$   
 $y = \cos(10) \cdot 100 = 98.48$

8. In the given triangles, use trigonometry to find the values of  $x$  and  $y$  to the nearest tenth. Check your answers by other means.

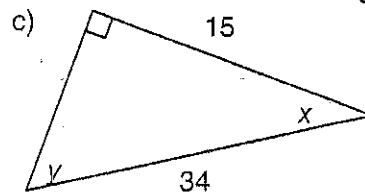
$$\begin{aligned}\sin X &= \frac{7}{13} \\ X &= 32.6 \\ \cos Y &= \frac{7}{13} \\ Y &= 57.4\end{aligned}$$



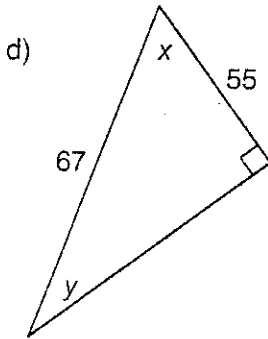
$$\begin{aligned}\tan X &= \frac{78}{8} \\ X &= 84.1 \\ \tan Y &= \frac{8}{78} \\ Y &= 5.9\end{aligned}$$



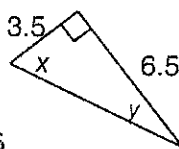
$$\begin{aligned}\cos X &= \frac{15}{34} \\ X &= 63.8 \\ \sin Y &= \frac{15}{34} \\ Y &= \sin^{-1}\left(\frac{15}{34}\right) = 26.2\end{aligned}$$



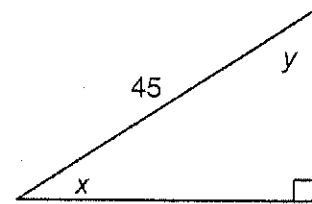
$$\begin{aligned}\cos X &= \frac{55}{67} \\ X &= 34.8 \\ \sin Y &= \frac{55}{67} \\ Y &= 55.2\end{aligned}$$



$$\begin{aligned}\tan X &= \frac{6.5}{3.5} \\ X &= 61.7 \\ \tan Y &= \frac{3.5}{6.5} \\ Y &= 28.3\end{aligned}$$

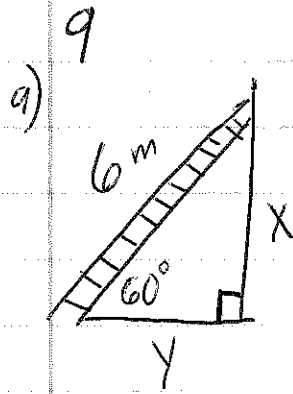


$$\begin{aligned}\sin X &= \frac{34}{45} \\ X &= 49.1 \\ \cos Y &= \frac{34}{45} \\ Y &= 40.9\end{aligned}$$



9. Use trigonometry to answer each of the following:

- A ladder, 6 m long, leans against a wall and makes an angle of  $60^\circ$  with the ground. How high up the wall does the ladder reach? How far from the wall is the foot of the ladder?
- A man starts at  $O$  and wishes to reach a point  $P$ , 300 m northeast of  $O$ . If he gets to  $P$  by first walking due north and then due east, how far will he have to walk in each direction?
- From the top of a building 20 m high, a man watches people walking along the street. If the angle of depression of the foot of a pedestrian is  $60^\circ$ , how far is the pedestrian from the foot of the building?
- The shadow of a pole is 4 m long when the angle of elevation of the sun is  $60^\circ$ . Find the length of the shadow when the angle of elevation of the sun is  $45^\circ$ .
- The length of the shadow of a 16 m tall tree is 8 m. What is the angle of elevation of the sun?
- A man whose eye is 1.5 m above the ground is standing 15 m from a tree which is 12 m high. What is the angle of elevation of the top of the tree from his eye? Give your answer correct to the nearest degree.

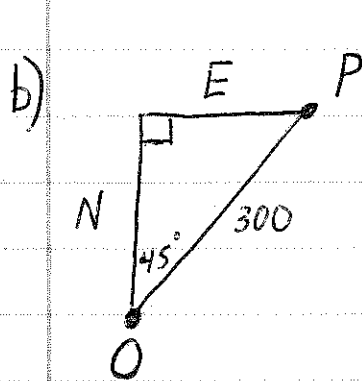


$$\sin 60^\circ = \frac{X}{6}$$

$$X = 5.196 \text{ m}$$

$$\cos 60^\circ = \frac{Y}{6}$$

$$Y = 3$$

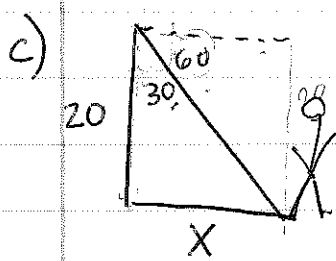


$$\sin 45^\circ = \frac{E}{300}$$

$$E = 300 \cdot \sin 45^\circ = 212.13$$

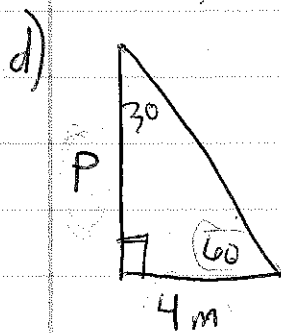
$$\cos 45^\circ = \frac{N}{300}$$

$$N = 212.13$$



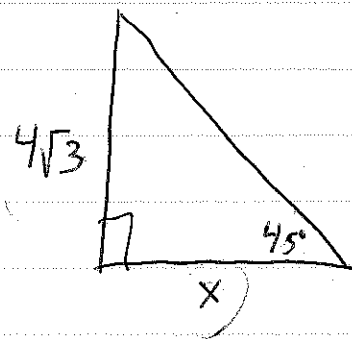
$$\tan 30^\circ = \frac{X}{20}$$

$$X = 11.5$$



~~tan~~

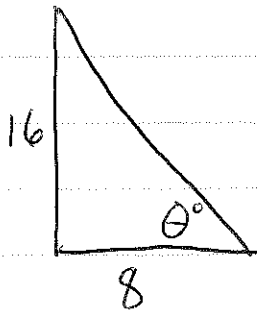
$$P = 4\sqrt{3}$$



$$X = 4\sqrt{3}$$

$$\approx 6.93$$

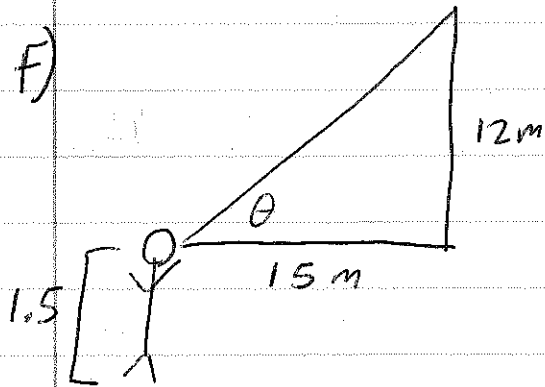
e)



$$\tan \theta = \frac{16}{8}$$

$$\tan^{-1}(2) = \boxed{63.4^\circ}$$

f)



$$\tan \theta = \frac{12}{15}$$

$$\tan^{-1}(4/5) = \boxed{38.7^\circ}$$