Name: Date:

# Linear Programming

TO SOLVE A LINEAR PROGRAMMING PROBLEM:

1.

2.

3.

4.

**Example 1 (maximum):** Determine the values of x and y that will maximize the objective function $P = 20x + 30y$ under the constraints:

$$\begin{cases} x + 2y \le 8 \\ 3x + 2y \le 12 \\ x \ge 0, y \ge 0 \end{cases}$$

Follow the steps!

Graph the system of linear inequalities. Determine the vertices.

In order to determine the vertex C, solve the system containing the equations that meet at C.

Now, test all "corner points" in the objective function.

<u>Vertex</u> <u>Value in Objective Function P = 20x + 30y</u>

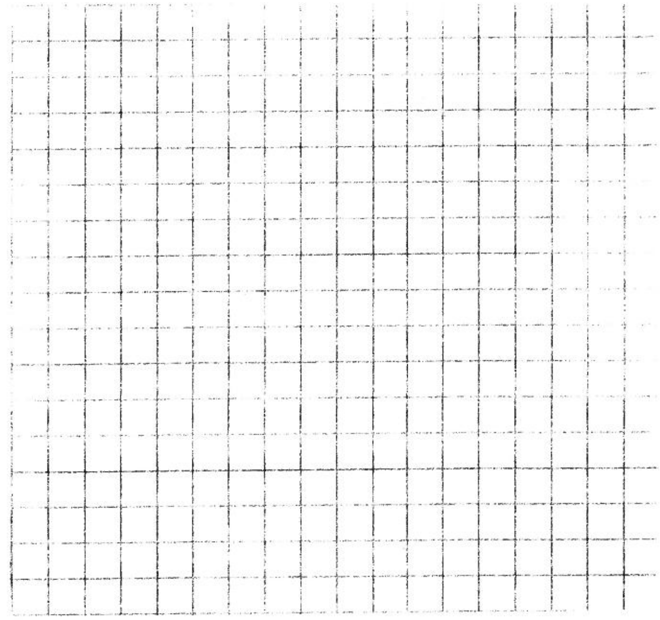
The maximum value of P is and occurs at the point ( , ).

**Example 2 (minimum):** Determine the value of x and y that will minimize the objective function $C = 5x + 15y$ under the constraints:

$$\begin{Bmatrix} 10x + 30y \geq 90 \\ 200x + 100y \geq 800 \\ x \geq 0, y \geq 0 \end{Bmatrix}$$

Use the same steps as before.

Graph the system of linear inequalities. Determine the vertices.



In order to determine the vertex F, solve the system containing the equations that meet at F.

Now, test all "corner points" in the objective function.

| Vertex | Value in Objective Function $C = 5x + 15y$ |
|---|---|

The minimum value of C is and occurs at

**Example 3:** Flywithus Airlines is updating its security system at a major airport. The budget for new metal detectors is $75,000. The airline has a maximum of 18 security guards available for each shift. There are two types of metal detectors available. Unit A costs $5000, requires one security guard, and can process 300 people per hour. Unit B costs $7500, requires two security guards, and can process 500 people per hour. Since Unit B has a better reliability record, the purchasing agent has mandated that at least four units must be type B. Determine the number of units of each type that should be purchased to maximize the number of people processed.

| | Unit A | Unit B | Limits |
|---|---|---|---|
| Number of Units | x | y | |
| Number of Guards | | | |
| Cost | | | |
| People Processed | | | |
| Minimum Required | | | |

Since the airline wants to maximize the number of people processed, the objective function is
P = , with the constraints:

Determine the feasible region and find the coordinates of the vertices.

Substitute each vertex in the objective function to determine the maximum value.

Vertex Value in Objective Function P= :

**Example 4:** A nutritionist is requested to devise a formula for a base for an instant breakfast meal. The breakfast must contain at least 12g of protein and 8g of carbohydrates. A tablespoon of protein powder made from soybeans has 5g of protein and 2g of carbohydrates. A tablespoon of protein powder made from milk solids has 2 g of protein and 4g of carbohydrates. Soybean protein powder costs \$0.70 per tablespoon, whereas milk protein powder costs \$0.30 per tablespoon. Determine the number of tablespoons for each type of protein powder that should be used as the base for this breakfast to meet the given requirements and minimize the cost.

Let x represent the number of tablespoons of soybean protein powder and let y represent the number of tablespoons of milk protein powder. Make a table to organize the given information.

| | Soybean | Milk | Requirements |
|---|---|---|---|
| Number of tbsp. | x | y | |
| Protein | | | |
| Carbohydrates | | | |
| Cost | | | |

Determine the feasible region and find the coordinates of the vertices.

Test each vertex in the objective function to determine the minimum value.

Vertex　　　　Value in Objective Function C=____________.

**Example 5:** A total of 44 planes were available on a given day during an airlift in World War II. Some planes were large and some were small. The large planes required four- person crews and the small planes required two- person crews, selected from a total of 128 crew members. A large plane could carry 30,000 cubic feet of cargo, and a small plane could carry 20,000 cubic feet of cargo. How many planes of each type would be necessary to maximize the cargo space?

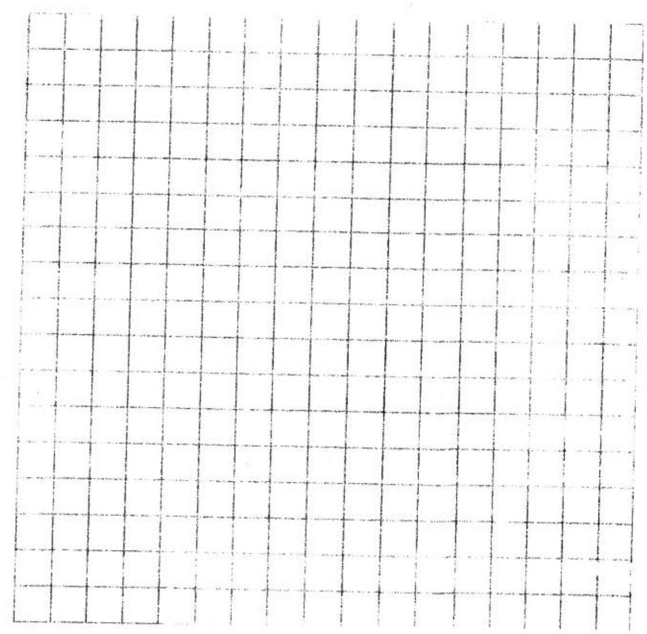
Let x represent the number of large planes and y the number of small planes. Fill in the table to organize the data.

| | Large | Small | Limits |
|---|---|---|---|
| Number of planes | | | |
| Number of crew | | | |
| Volume of cargo | | | |

Objective Function:

Constraints:

Graph the feasible region and find the coordinates of the vertices.



Test each vertex in the objective function.

Vertex　　　　　　　Value in Objective Function V= ________________:

Name: Date:

## Linear Programming

SOLVE A LINEAR PROGRAMMING PROBLEM:

1. Write the objective function (expression to min or max)
2. Write and graph the constraints (inequalities)
3. Find the vertices of the feasible region.
4. Plug the vertices into the O.F., determine optimal solution.

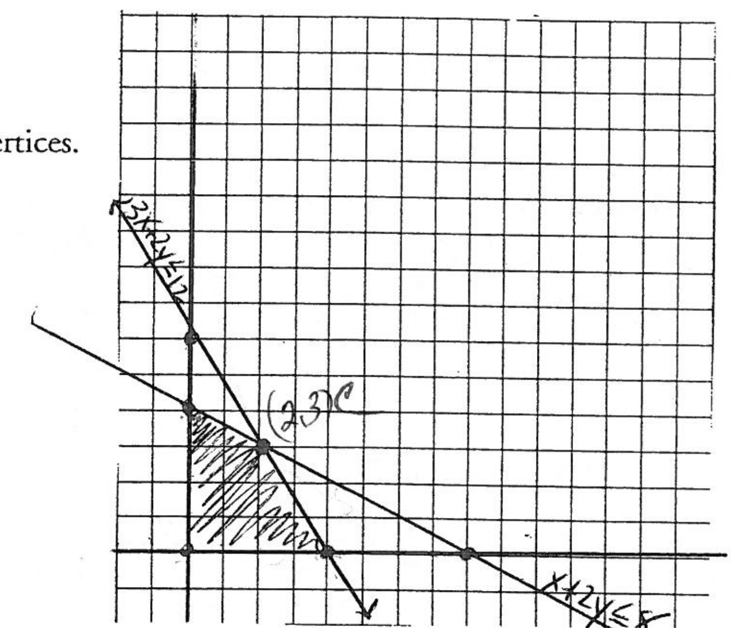
**Example 1 (maximum):** Determine the values of x and y that will maximize the objective function $P = 20x + 30y$ under the constraints:

$$\begin{cases} x + 2y \le 8 \\ 3x + 2y \le 12 \\ x \ge 0, y \ge 0 \end{cases}$$

Follow the steps!

Graph the system of linear inequalities. Determine the vertices.

(0,0) (0,4) (4,0) (2,3)




In order to determine the vertex C, solve the system containing the equations that meet at C.

$$x + 2y = 8$$
$$3x + 2y = 12$$
$$-2x = -4 \quad x = 2 \quad y = 3$$

Now, test all "corner points" in the objective function.

| Vertex | Value in Objective Function $P = 20x + 30y$ |
| --- | --- |
| (0,0) | $P = 20(0) + 30(0) = 0$ |
| (0,4) | $P = 20(0) + 30(4) = 120$ |
| (4,0) | $P = 20(4) + 30(0) = 80$ |
| (2,3) | $P = (20)(2) + 30(3) = 130$ |

The maximum value of P is 130 and occurs at the point (2, 3).

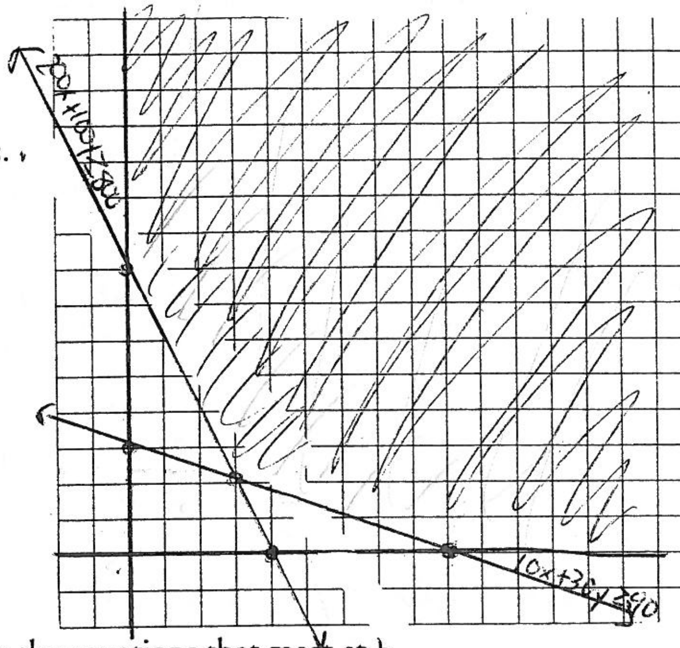
**Example 2 (minimum):** Determine the value of x and y that will minimize the objective function C = 5x + 15y under the constraints:

$$\begin{cases} 10x + 30y \geq 90 \\ 200x + 100y \geq 800 \\ x \geq 0, y \geq 0 \end{cases}$$

Use the same steps as before.

Graph the system of linear inequalities. Determine the vertices.

(0,8)
(9,0)
(3,2)



In order to determine the vertex F, solve the system containing the equations that meet at F

$$\begin{matrix} 10x+30y=90 \\ 200x+100y=800 \end{matrix} \rightarrow \begin{matrix} x+3y=9 \\ 2x+y=8 \end{matrix} \rightarrow \begin{matrix} -2x-6y=-18 \\ 2x+y=8 \\ \hline -5y=-10 \\ y=2 \end{matrix}$$

$x+6=9$
$x=3$

Now, test all "corner points" in the objective function.

| <u>Vertex</u> | <u>Value in Objective Function C = 5x + 15y</u> |
|---|---|
| (0,8) | $5(0)+15(8)=120$ |
| (9,0) | $5(9)+15(0)=45$ |
| (3,2) | $5(3)+15(2)=45$ |

The minimum value of C is 45 and occurs at the line segment with end points (9,0) and (3,2)

$y=-1/3x+3$
where $3 \leq x \leq 9$ and $0 \leq y \leq 2$

Example 3: Flywithus Airlines is updating its security system at a major airport. The budget for new metal detectors is \$75,000. The airline has a maximum of 18 security guards available for each shift. There are two types of metal detectors available. Unit A costs \$5000, requires one security guard, and can process 300 people per hour. Unit B costs \$7500, requires two security guards, and can process 500 people per hour. Since Unit B has a better reliability record, the purchasing agent has mandated that at least four units must be type B. Determine the number of units of each type that should be purchased to maximize the number of people processed.

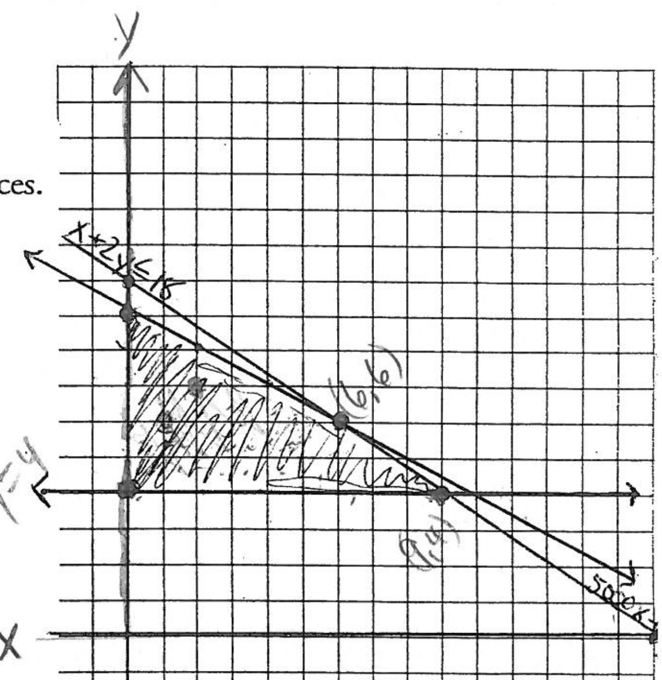
| | Unit A | Unit B | Limits |
|---|---|---|---|
| Number of Units | x | y | |
| Number of Guards | 1 | 2 | 18 |
| Cost | 5000 | 7500 | 75,000 |
| People Processed | 300 | 500 | objective Function. |
| Minimum Required | 0 | 4 | |

Since the airline wants to maximize the number of people processed, the objective function is
P = $300x + 500y$, with the constraints:

$$\begin{cases} x + 2y \le 18 \\ 5000x + 7500y \le 75{,}000 \quad x + 1.5y \le 15 \\ x \ge 0 \\ y \ge 4 \end{cases}$$

Determine the feasible region and find the coordinates of the vertices.

(0,4) (0,9) (9,4) (6,6)




Substitute each vertex in the objective function to determine the maximum value.

| Vertex | Value in Objective Function P= $300x+500y$ |
|---|---|
| (0,4) | $300(0)+500(4)=2000$ |
| (0,9) | $300(0)+500(9)=4{,}500$ |
| (9,4) | $300(9)+500(4)=4{,}700$ |
| (6,6) | $300(6)+500(6)=4{,}800$ * |

6 → Unit A
6 → Unit B

6 of A and 6 of B, they will process a maximum of 4,800 people.

**Example 4:** A nutritionist is requested to devise a formula for a base for an instant breakfast meal. The breakfast must contain at least 12g of protein and 8g of carbohydrates. A tablespoon of protein powder made from soybeans has 5g of protein and 2g of carbohydrates. A tablespoon of protein powder made from milk solids has 2 g of protein and 4g of carbohydrates. Soybean protein powder costs \$0.70 per tablespoon, whereas milk protein powder costs \$0.30 per tablespoon. Determine the number of tablespoons for each type of protein powder that should be used as the base for this breakfast to meet the given requirements and minimize the cost.

Let x represent the number of tablespoons of soybean protein powder and let y represent the number of tablespoons of milk protein powder. Make a table to organize the given information.

| | Soybean | Milk | Requirements |
|---|---|---|---|
| Number of tbsp. | x | y | |
| Protein | 5 | 2 | 12 |
| Carbohydrates | 2 | 4 | 8 |
| Cost | .70 | .30 | |

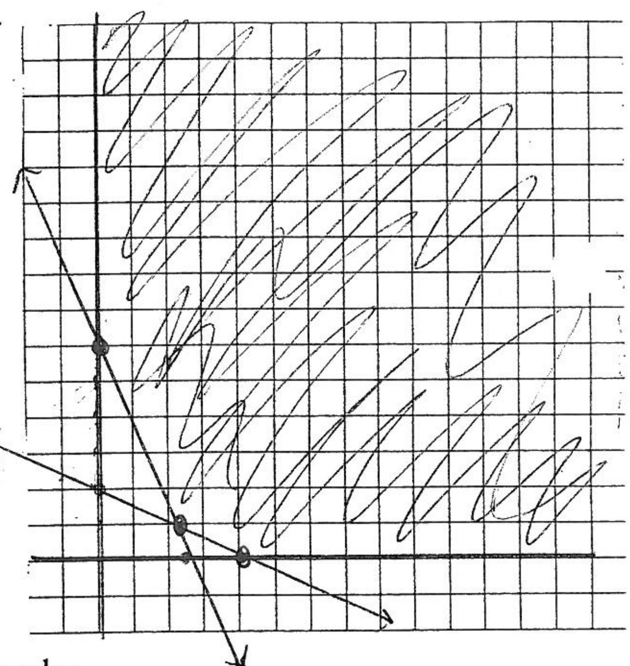
Determine the feasible region and find the coordinates of the vertices.

$$\begin{cases} 5x+2y\ge 12 \\ 2x+4y\ge 8 \\ x\ge 0 \\ y\ge 0 \end{cases}$$

(0,6)
(4,0)
(2, 1)

$(5x+2y=12)-2$
$2x+4y=8$
$-10x-4y=-24$
$-8x=-16$
$x=2$

$10+2y=12$
$2y=2$
$y=1$



Test each vertex in the objective function to determine the minimum value.

| Vertex | Value in Objective Function C= $.70x+.30y$ |
|---|---|
| (0,6) | $.7(0)+.3(6)=1.80$ |
| (4,0) | $.7(4)+.3(0)=2.80$ |
| (2,1) | $.7(2)+.3(1)=1.70$ * |

2 tbsp from Soybean
1 tbsp from Milk

**Example 6**: Every day, Shannon Miller needs a dietary supplement of 4 mg of Vitamin A, 11mg of Vitamin B, and 100 mg of Vitamin C. Either of two brands of vitamin pills can be used: Brand X at \$0.06 per pill or Brand Y or \$0.08 per pill. A Brand X pill supplies 2 mg of Vitamin A, 3 mg of Vitamin B, and 25 mg of Vitamin C. Likewise, a Brand Y pill supplies 1, 4, and 50 mg of vitamins A, B, and C, respectively. How many pills of each brand should she take each day in order to satisfy the minimum daily need most economically?

| | Brand X | Brand y | Limits |
|---|---|---|---|
| Vitamin A | 2 | 1 | 4 |
| Vitamin B | 3 | 4 | 11 |
| Vitamin C | 25 | 50 | 100 |
| Cost | .06 | .08 | |

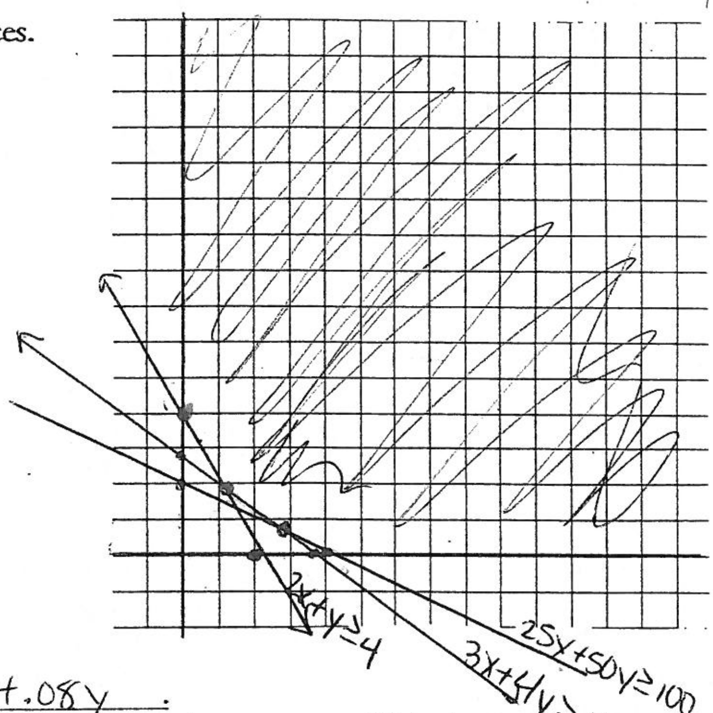
Let x= Brand X

Let y= Brand Y

Objective Function: $D = .06x + .08y$

Constraints: $\begin{cases} 2x+y \geq 4 \\ 3x+4y \geq 11 \\ 25x+50y \geq 100 \\ x \geq 0,\ y \geq 0 \end{cases}$

Graph the feasible region and find the coordinates of the vertices.




$3x+4y=11$

$2x+y=4$

$y=4-2x$

$3x+4(4-2x)=11$

$3x+16-8x=11$

$-5x=-5$

$x=1 \quad y=2$

$25x+50y=100$

$3x+4y=11$

$x+2y=4$

$3(-2y+4)+4y=11$

$-6y+12+4y=11$

$12-2y=11$

$y=\frac{1}{2}$

$x=3$

Test each vertex in the objective function.

| Vertex | Value in Objective Function V= $.06x+.08y$ |
|---|---|
| (1,2) | $.06(1)+.08(2) = .22$ |
| (3,1/2) | $.06(3)+.08(1/2) = .22$ |
| (0,4) | $.06(0)+.08(4) = .32$ |
| (4,0) | $.06(4)+.08(0) = .24$ |

$\frac{3/2}{-2}$

Line segment between (1,2) (3,1/2)

$y = -\frac{3}{4}x + 2\frac{3}{4} \quad 1 \leq x \leq 3$

**Example 5:** A total of 44 planes were available on a given day during an airlift in World War II. Some planes were large and some were small. The large planes required four- person crews and the small planes required two- person crews, selected from a total of 128 crew members. A large plane could carry 30,000 cubic feet of cargo, and a small plane could carry 20,000 cubic feet of cargo. How many planes of each type would be necessary to maximize the cargo space?

Let x represent the number of large planes and y the number of small planes. Fill in the table to organize the data.

| | Large | Small | Limits |
|---|---|---|---|
| Number of planes | X | Y | 44 |
| Number of crew | 4 | 2 | 128 |
| Volume of cargo | 30,000 | 20,000 | |

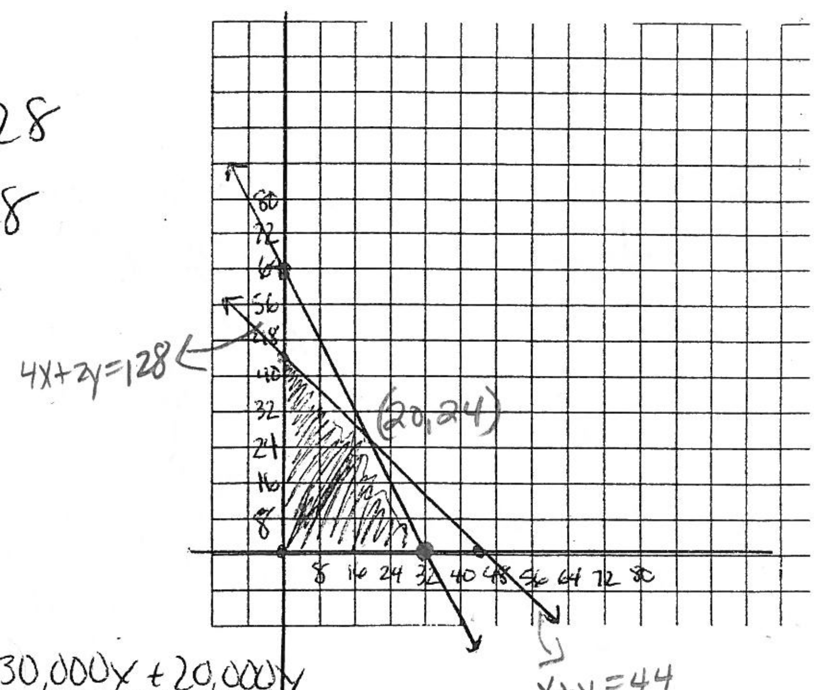
Objective Function: $30{,}000x + 20{,}000y = V$

Constraints: $\left\{\begin{array}{l} x + y \le 44 \\ 4x + 2y \le 128 \\ x \ge 0,\ y \ge 0 \end{array}\right\}$

Graph the feasible region and find the coordinates of the vertices.

(0,0)
(0,44)
(32,0)
(20,24)

$x = 44 - y$
$4(44 - y) + 2y \le 128$
$176 - 4y + 2y = 128$
$176 - 2y = 128$
$y = 24$
$x = 20$




Test each vertex in the objective function.

| Vertex | Value in Objective Function V= $30{,}000x + 20{,}000y$ |
|---|---|
| (0,0) | $30{,}000(0) + 20{,}000(0) = 0$ |
| (0,44) | $30{,}000(0) + 20{,}000(44) = 880{,}000$ |
| (32,0) | $30{,}000(32) + 20{,}000(0) = 960{,}000$ |
| (20,24) | $30{,}000(20) + 20{,}000(24) = 1{,}080{,}000$ ☆ ↳ $ft^3$ |

20 Large 24 Small