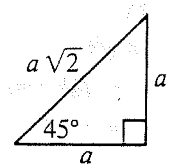
NAME ______________________________ DATE ______________

8–4 Special Right Triangles

Objective: Determine the lengths of two sides of a 45°-45°-90° or a 30°-60°-90° triangle when the length of the third side is known.

> 45°-45°-90° Theorem **In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as a leg.**

A 45°-45°-90° triangle is an isosceles right triangle with congruent legs. If the length of a leg is a, then the length of the hypotenuse is $a\sqrt{2}$.



Example 1 Given the length of the legs, find the length of the hypotenuse of each 45°-45°-90° triangle.

a. 5 **b.** $3\sqrt{2}$ **c.** $5\sqrt{6}$

Solution

a. $5\sqrt{2}$ **b.** $3\sqrt{2} \cdot \sqrt{2} = 6$ **c.** $5\sqrt{6} \cdot \sqrt{2} = 5\sqrt{12} = 10\sqrt{3}$

Example 2 Given the length of the hypotenuse, find the length of the legs of each 45°-45°-90° triangle.

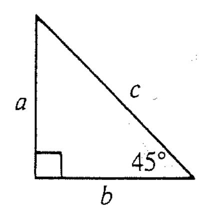
a. $8\sqrt{2}$ **b.** 10 **c.** $4\sqrt{3}$

Solution

a. $\frac{8\sqrt{2}}{\sqrt{2}} = 8$ **b.** $\frac{10}{\sqrt{2}} = \frac{10 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$ **c.** $\frac{4\sqrt{3}}{\sqrt{2}} = \frac{4\sqrt{3} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{4\sqrt{6}}{2} = 2\sqrt{6}$

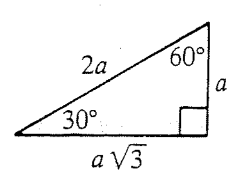
Complete the table.

	1.	2.	3.	4.	5.	6.	7.	8.
a	3	?	?	$\frac{1}{2}$	?	?	?	?
b	?	?	$6\sqrt{2}$	?	?	$5\sqrt{3}$	?	?
c	?	$5\sqrt{2}$	?	?	$8\sqrt{6}$	?	12	9



> 30°-60°-90° Theorem **In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.**

In a 30°-60°-90° triangle, the shorter leg is opposite the 30° angle and the longer leg is opposite the 60° angle. The theorem says if the shorter leg has length a, then the hypotenuse has length $2a$ and the longer leg has length $a\sqrt{3}$.



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8–4 Special Right Triangles *(continued)*

Example 3 Using the side given, find the other two sides of each 30°-60°-90° triangle.

a. shorter leg: $8\sqrt{3}$ **b.** hypotenuse: 12 **c.** longer leg: $\sqrt{6}$

Solution

a. hyp.: $= (\text{shorter leg}) \cdot 2 = 8\sqrt{3} \cdot 2 = 16\sqrt{3}$

longer leg $= (\text{shorter leg}) \cdot \sqrt{3} = 8\sqrt{3} \cdot \sqrt{3} = 24$

b. shorter leg $= \frac{\text{hyp.}}{2} = \frac{12}{2} = 6$

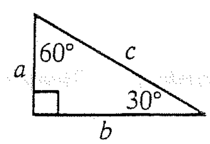
longer leg $= (\text{shorter leg}) \cdot \sqrt{3} = 6\sqrt{3}$

c. shorter leg $= \frac{\text{longer leg}}{\sqrt{3}} = \frac{\sqrt{6}}{\sqrt{3}} = \frac{\sqrt{2} \cdot \sqrt{3}}{\sqrt{3}} = \sqrt{2}$

hyp. $= (\text{shorter leg}) \cdot 2 = 2\sqrt{2}$

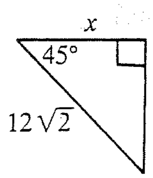
Complete the table.

	9.	10.	11.	12.	13.	14.	15.	16.
a	10	?	?	$6\sqrt{2}$	?	?	?	?
b	?	?	$5\sqrt{3}$	?	12	?	15	?
c	?	24	?	?	?	$7\sqrt{3}$	?	$2\sqrt{2}$



Find the value of x.

17.



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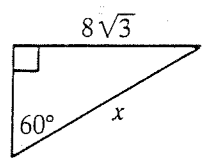

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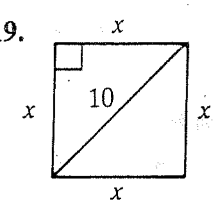

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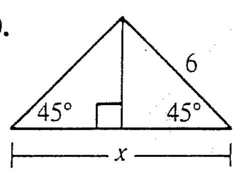

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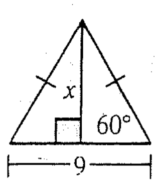

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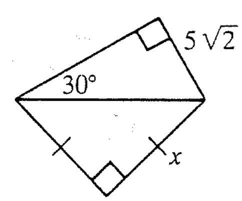

Find the values of x and y.

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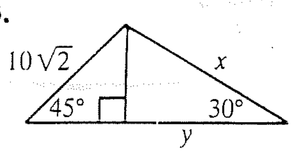

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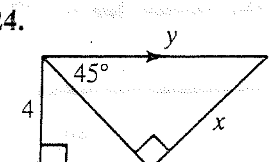

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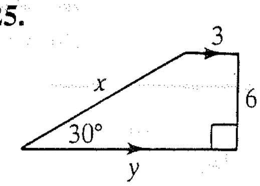

26. Find the perimeter of a square if a diagonal has length 12.

27. Find the perimeter of an equilateral triangle if an altitude has length $7\sqrt{3}$.

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