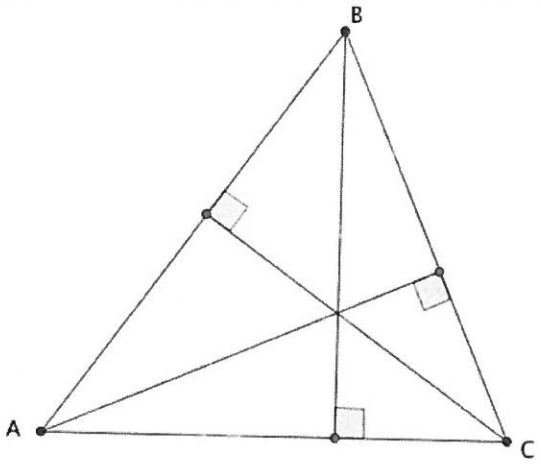
Area of a Triangle: (A small extension of $\frac{1}{2} \cdot \text{base} \cdot \text{height}$)



Suppose that in the triangle at left you know all three angles and the measures of all three sides. You don't immediately know the measure of any of the altitudes but you could definitely find them.

Question: How do you calculate the *height* in the ambiguous case? How is that related to this?

Question: Summarize the method of finding the area of a triangle in which you know two sides and the included angle.

Find the area of each possible triangle. (Remember to check the case and how many triangles there could be…)

$\triangle JKL$: $J = 25°$, $k = 25$, $l = 12$ 63.4	$\triangle MNP$: $M = 25°$, $N = 34°$, $p = 12$ 19.9
$\triangle QRS$: $Q = 25°$, $q = 25$, $r = 60$ No 	$\triangle TUV$: $U = 43°$, $u = 16$, $t = 12$ 92.2
$\triangle WXY$: $Y = 32°$, $w = 25$, $y = 16$ 199.9	$\triangle ABZ$: $B = 105°$, $b = 25$, $a = 15$ 119.5

Area of a Triangle: *Heron's Formula*

To find the area of a triangle for which we know all of the sides we do not actually need to find the height. Instead we can use *Heron's Formula*:

Let ΔABC have sides a, b, and c and let $s = \dfrac{a+b+c}{2}$ (the semi-perimeter), then the area of the triangle is given by

$$\boxed{\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}}$$

This formula is very useful in finding the areas of triangles that you have already solved.

Question: Use Heron's Formula to find the area of these triangles:

a. $a = 3,\ b = 4,\ c = 5$

6

b. $a = 2.9,\ b = 3.7,\ c = 4.5$

5.3

The problems below are for practice. Completely solve any possible triangles and then find their areas. Use the appropriate Law, the most convenient approach, and show your work and answers in an organized way.

1. ΔABC : $A = 30°,\ b = 45,\ c = 20$

$a = 29.4$
$B = 130°$
$C = 20°$
$A = 225$

2. ΔABC : $a = 20,\ b = 60,\ c = 45$

$A = 14.6°$
$B = 130.8°$
$C = 34.6°$
$A = 341$

3. ΔABC : $A = 40°,\ B = 70°,\ c = 30$

$a = 20.5$
$b = 30$
$C = 70°$
$A = 289.3$

4. ΔABC : $A = 20°,\ B = 60°,\ a = 90$

$b = 227.9$
$C = 259.1$
$C = 100°$
$A = 10094.2$

5. ΔABC : $A = 35°,\ a = 20,\ b = 30$

$C = 34.8$
$B = 59°$
$C = 86°$
$A = 294$

6. ΔABC : $B = 22°,\ a = 52,\ c = 46$

$A = 96°$
$C = 61°$
$b = 19.6$
$A = 448$

7. ΔABC : $a = 15,\ b = 60,\ c = 74$

$A = 4.6°$
$B = 18.8°$
$C = 156.5°$
$A = 179.2$

8. ΔABC : $A = 25°,\ C = 22°,\ b = 65$

$a = 37.5$
$C = 33.3$
$B = 133°$
$A = 457.3$

9. ΔABC : ~~A~~ $= 50°,\ B = 43°, b = 12$

Typo A
$a = 13.5$
$C = 17.6$
$C = 87°$
$A = 80.7$

10. ΔABC : $A = 17°,\ a = 20,\ c = 15$

$b = 32.9$
$B = 150.3°$
$C = 12.7°$
$A = 74.2$

11. ΔABC : $B = 107°,\ b = 2,\ a = 15$

No Sol.

12. ΔABC : $C = 88°,\ a = 22,\ c = 5$

No Sol.