

Name: _____
 Period: _____

Date: _____
 Pre-Calculus: Conics Introduction

Classify each of the following conic sections. On the line provided, write whether the given conic is a circle, ellipse, hyperbola, or parabola.

1. $3x^2 + y^2 - 6x - 3 = 0$ Ellipse
2. $2x^2 - y^2 + 12x + 14 = 0$ Hyperbola
3. $4y^2 - 16x - 12y - 23 = 0$ Parabola
4. $2x^2 + 2y^2 - 3y - 1 = 0$ Circle
5. $2x^2 - 3y = 0$ parabola
6. $x^2 + y^2 + 2x - 2y - 2 = 0$ Circle
7. $2x^2 - 2y^2 + 60y - 63 = 0$ Hyperbola
8. $4x^2 + 9y^2 - 16x + 72y + 124 = 0$ Ellipse
9. $16x^2 + 4y^2 + 64x - 12y + 57 = 0$ Ellipse
10. $y^2 - 4y - 5x - 1 = 0$ parabola
11. $25x^2 - 9y^2 + 150x + 36y - 36 = 0$ Hyperbola

② Sketch pad

③

Let's take this equation in *general* form, and put it in standard form

$$4x^2 + 25y^2 - 24x - 64 = 0$$

$$4x^2 - 24x + 25y^2 = 64$$

$$4(x^2 - 6x + 9) + 25y^2 = 64 + 36$$

$$\frac{4(x-3)^2}{4 \cdot 25} + \frac{25y^2}{4 \cdot 25} = \frac{100}{4 \cdot 25}$$

$$\frac{(x-3)^2}{25} + \frac{y^2}{4} = 1$$

Definitions

- **Ellipse** – If $F_1(c, 0)$ and $F_2(-c, 0)$ are two fixed points in a plane and a is a constant, $0 < c < a$, then the set of all points P in the plane such that $PF_1 + PF_2 = 2a$ is an ellipse
- **Focus Point** (plural – foci) – the fixed points F_1 and F_2
- **Major Axis** – The longest diameter of an ellipse
- **Minor Axis** – The shortest diameter of an ellipse

Algebraic Definition of Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \text{ where } b^2 = a^2 - c^2$$

Center $\rightarrow (h, k)$

$2a =$ Horizontal ~~Asymptote~~ AXIS

$2b =$ Vertical ~~Asymptote~~ AXIS

$c =$ Horizontal distance focus to center.

④ Do two as a class.

⑤ Write the Equation

Writing Equations for Ellipses

1. Write the equation for an ellipse with center (3,0), vertical major axis of length 10 and minor axis of length 6.

$$\frac{(X-3)^2}{9} + \frac{Y^2}{25} = 1$$

$(3, 5)$
 $(3, -5)$
 $(3, 4)$
 $(3, -4)$
 $(3, 0)$
 $a=5$
 $b=3$
 $c=5$

2. Write the equation for an ellipse with center (-1, -1) with horizontal major axis of length 16 and minor axis of length 4. $b=2$

$$\frac{(X+1)^2}{64} + \frac{(Y+1)^2}{4} = 1$$

$a=8$

3. Write the equation for an ellipse with foci (-1,0) and (1,0) and major axis of length 4.

$$\frac{X^2}{4} + \frac{Y^2}{3} = 1$$

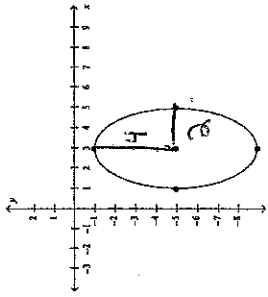
$(-1, 0)$ $(1, 0)$ $(1, 0)$ $(2, 0)$
 $b = \sqrt{3}$
 $a = 2$

4. Write the equation for an ellipse with vertices at (6,0) and (-6,0) and foci at (2,0) and (-2,0)

$$\frac{X^2}{36} + \frac{Y^2}{32} = 1$$

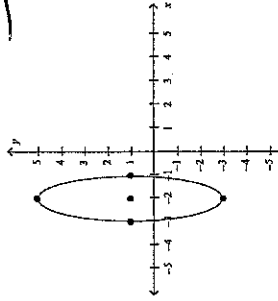
$(-6, 0)$ $(2, 0)$ $(0, 0)$ $(2, 0)$ $(6, 0)$
 $b = \sqrt{32}$

5.



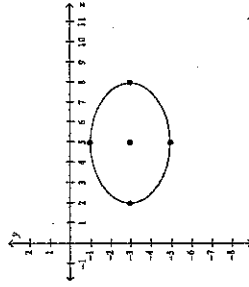
$$\frac{(X-3)^2}{4} + \frac{(Y+5)^2}{16} = 1$$

6.



$$\frac{(X+2)^2}{1} + \frac{(Y-1)^2}{16} = 1$$

7.



$$\frac{(X-5)^2}{9} + \frac{(Y+3)^2}{4} = 1$$

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 Pre-Calculus: *Ellipse Classwork Worksheet*

1. Find the center, vertices, covertices, foci, length of major and minor axes, and sketch the graph.

a. $\frac{(x-4)^2}{9} + \frac{(y+3)^2}{25} = 1$

center: $(4, -3)$

major axis: $X=4$

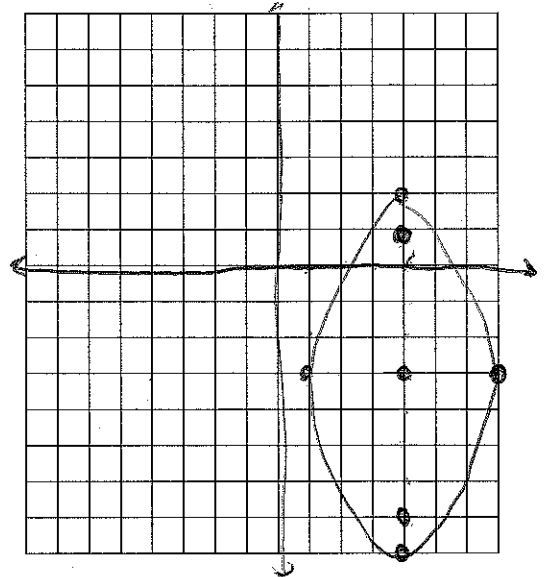
minor axis: $Y=-3$

vertices: $(4, 2)$ $(4, -8)$

covertices: $(1, -3)$ $(7, -3)$

foci: $(4, 1)$ $(4, -7)$

$$\begin{aligned} a^2 &= 25 - c^2 \\ -16 &= -c^2 \\ 4 &= c \end{aligned}$$



b. $x^2 + 5y^2 - 8x - 30y - 39 = 0$ $x^2 - 8x + 5y^2 - 30y = 39$

center: $(4, 3)$

major axis: $Y=3$

minor axis: $X=4$

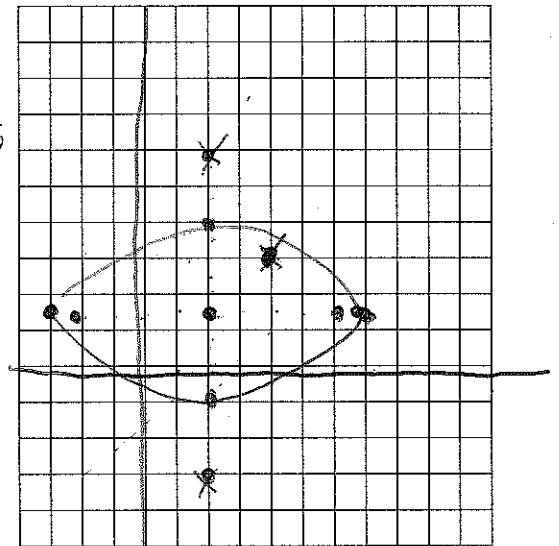
vertices: $(14, 3)$ $(-6, 3)$

covertices: $(4, 3+\sqrt{20})$ $(4, 3-\sqrt{20})$

foci: $(4-\sqrt{80}, 3)$ $(4+\sqrt{80}, 3)$

$$\begin{aligned} (x^2 - 8x + 16) + 5(y^2 - 6y + 9) &= 39 + 45 + 16 \\ \frac{(x-4)^2}{100} + \frac{5(y-3)^2}{100} &= 100 \end{aligned}$$

$$\begin{aligned} 20 &= 100 - c^2 \\ -80 &= -c^2 \\ c &= \sqrt{80} \end{aligned}$$



c. $9x^2 + 4y^2 + 54x + 8y + 49 = 0$

center: $(-3, -1)$

major axis: $X=-3$

minor axis: $Y=-1$

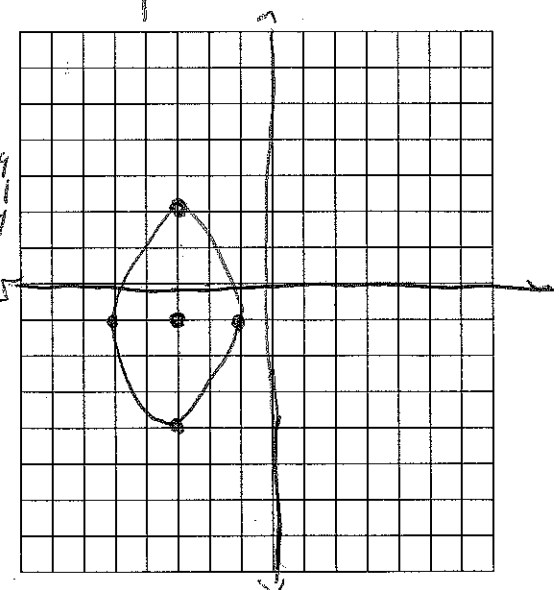
vertices: $(2, -1)$ $(-8, -1)$

covertices: $(-3, -1+\sqrt{5})$ $(-3, -1-\sqrt{5})$

foci: $(-5, -1+\sqrt{5})$ $(-1, -1-\sqrt{5})$

$$\begin{aligned} 9x^2 + 54x + 4y^2 + 8y &= -49 \\ 9(x^2 + 6x + 9) + 4(y^2 + 2y + 1) &= -49 + 81 + 4 \\ \frac{9(x+3)^2}{36} + \frac{4(y+1)^2}{36} &= \frac{36}{36} \\ \frac{(x+3)^2}{4} + \frac{(y+1)^2}{9} &= 1 \end{aligned}$$

$$9 - 4 = \sqrt{5}$$



d. $25x^2 + y^2 - 300x + 8y + 891 = 0$

center: $(6, -4)$

major axis: $X = 6$

minor axis: $Y = -4$

vertices: $(6, 1), (6, -9)$

covertices: $(7, -4), (5, -4)$

foci: $(6, -4 + \sqrt{24}), (6, -4 - \sqrt{24})$

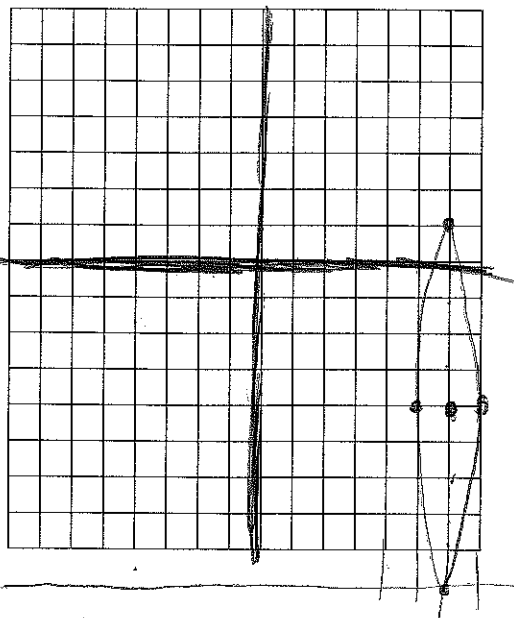
$$25x^2 - 300x + y^2 + 8y = -891$$

$$25(x^2 - 12x + 36) + y^2 + 8y + 16 = -891 + 900 + 16$$

$$\frac{25(x-6)^2}{25} + \frac{(y+4)^2}{25} = \frac{25}{25}$$

$$\frac{(x-6)^2}{1} + \frac{(y+4)^2}{25} = 1$$

$$1 - 25 = \sqrt{24}$$



2. Write the equation of an ellipse given the provided information.

a. Vertices at $(3, 1)$ and $(3, 9)$. Minor axis of length 6. \rightarrow ~~6~~ $a=3$

Major Axis \rightarrow 8 ~~6~~ $b=4$

center $(3, 5)$

$$\frac{(x-3)^2}{9} + \frac{(y-5)^2}{16} = 1$$

b. Vertices at $(5, 0)$ and $(5, 12)$. Covertices at $(0, 6)$ and $(10, 6)$

$$b = 6$$

$$a = 5$$

center $(5, 6)$

$$\frac{(x-5)^2}{25} + \frac{(y-6)^2}{36} = 1$$

c. Foci are at $(1, 0)$ and $(-1, 0)$ and the length of the major axis is 4.

center $(0, 0)$

$$a = 2$$

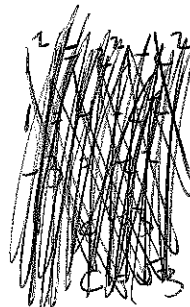
$$c = 1$$

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$b^2 = 2^2 - 1^2$$

$$b^2 = 3$$

$$b = \sqrt{3}$$



d. Vertical major axis is 10, minor axis is 6, and center is $(3, 0)$.

$$a = 5 \quad b = 3$$

$$\frac{(x-3)^2}{9} + \frac{y^2}{25} = 1$$