

# Hints

Period: \_\_\_\_\_ Date: \_\_\_\_\_

## Chapter 5 Practice Problems

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\sin\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1-\cos A}{2}}$$

$$\cos(2A) = 2 \cos^2 A - 1$$

$$\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cos(2A) = 1 - 2 \sin^2 A$$

$$\cos\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1+\cos A}{2}}$$

Prove the identity.

$$1) \frac{\cos^4 x - \sin^4 x}{\sin^2 x} = \cot^2 x - 1$$

Factor

$$\Rightarrow \frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{\sin^2 x}$$

$$\frac{\cos^2 x - \sin^2 x}{\sin^2 x}$$

$$\frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x} = \cot^2 x - 1 \quad \checkmark$$

$$2) \csc B - \sin B = \cot B \cos B$$

$$\frac{1}{\sin B} - \sin B = \frac{\cos B}{\sin B}, \frac{\cos B}{\sin B}$$

$$\frac{1 - \sin^2 B}{\sin B} = \frac{\cos^2 B}{\sin B}$$

$$\frac{1 - \sin^2 B}{\sin B} = \frac{\cos^2 B}{\sin B}$$

$$\frac{\cos^2 B}{\sin B} = \frac{\cos^2 B}{\sin B}$$

$$3) \cos\left(\frac{\pi}{2} + x\right) = -\sin x \quad \text{Use } \cos(A+B) \text{ formula}$$

$$\cos\frac{\pi}{2} \cos x - \sin\frac{\pi}{2} \sin x$$

$$0 \cdot \cos x - 1 \cdot \sin x$$

$$-\sin x \quad \checkmark$$

$$1 - 2 \sin^2 A$$

$$4) \frac{2 - 2\cos 2x}{\sin 2x} = \sec x (\csc x) - \cot x + \tan x$$

Use double angle formulas

$$\begin{aligned} \cancel{\frac{2 - 2(\cancel{2\cos^2 A} - 1)}{2\sin A \cos A}} &= \frac{1}{\cos A \sin A} - \frac{\cos A + \sin A}{\sin A \cos A} \\ \cancel{\frac{-4\cos^2 A - 2}{2\sin A \cos A}} &= \frac{1}{\cos A \sin A} - \frac{\cos^2 A + \sin^2 A}{\cos A \sin A} \\ \cancel{\frac{-2\cos A}{\sin A}} &= \frac{1 - \cos^2 A + \sin^2 A}{\cos A \sin A} \\ \cancel{\frac{2\sin A}{\cos A}} &= \frac{\sin^2 A + \sin^2 A}{\cos A \sin A} \\ \cancel{\frac{2\sin A}{\cos A}} &= \frac{2\sin A}{\cos A} = \frac{2\sin A}{\cos A} \end{aligned}$$

$$5) \text{Prove } (\cos x - \sin x)(\cos x + \sin x) = \cos 2x$$

FOIL & Double Angle formula

$$\cos^2 x - \sin^2 x$$

$$\cos 2x \quad \checkmark$$

Evaluate without a calculator.

$$6) \tan(165) \quad \tan(A + B)$$

$$\begin{aligned} \tan(135 + 30) &= \tan 135 + \tan 30 \\ \frac{\sqrt{3} - 1}{1 + \sqrt{3}/3} \cdot 3 &= -1 + \frac{\sqrt{3}/3}{1 - (-1)(\sqrt{3}/3)} \cdot \frac{\sqrt{3} - 3/2}{\sqrt{3}} \\ \frac{\sqrt{3} - 3}{3 + \sqrt{3}} \cdot \frac{\sqrt{3} - 3}{\sqrt{3}} &\rightarrow \frac{(-6)}{(-6)} = 1 \end{aligned}$$

$$8) \cos\left(\frac{\pi}{15}\right)\cos\left(\frac{\pi}{10}\right) - \sin\left(\frac{\pi}{10}\right)\sin\left(\frac{\pi}{15}\right)$$

$$\cos(A + B)$$

$$\cos\left(\frac{\pi}{15} + \frac{\pi}{10}\right) = \cos(12 + 18)$$

$$\cos 30$$

$$\frac{\sqrt{3}}{2}$$

$$7) \csc(15) \quad \text{use } \sin\left(\frac{A}{2}\right)$$

$$\begin{aligned} \sin\left(\frac{30}{2}\right) &= \pm \sqrt{\frac{1 - \cos 30}{2}} \text{ then flip at end} \\ &= \frac{1 - \sqrt{3}/2}{\sqrt{2}} \\ &= \frac{\sqrt{2} - \sqrt{3}}{\sqrt{4}} = \frac{\sqrt{2} - \sqrt{3}}{2} \end{aligned}$$

$$9) \cos^2 195 - \sin^2 195$$

$$\cos(2 \cdot 195)$$

$$\cos(390)$$

$$= \frac{\sqrt{3}}{2}$$

$\sin A$

$\sec A = -\frac{4}{5}$  with A in QIV and  $\cot B = -\frac{12}{5}$  with B in QII, find:

a.  $\cos(A-B)$

$$\begin{aligned} \cos A \cos B + \sin A \sin B \\ \left(\frac{3}{5}\right)\left(\frac{-9}{13}\right) + \left(-\frac{4}{5}\right)\left(\frac{12}{13}\right) \\ \frac{-15-48}{65} = -\frac{63}{65} \end{aligned}$$

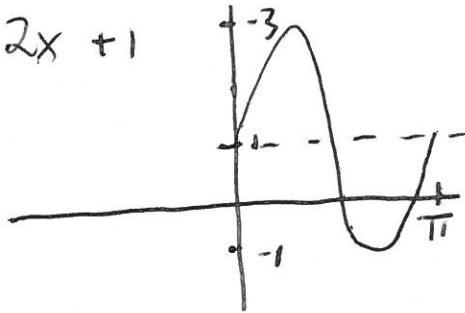
c.  $\cot(A-B)$

$$\begin{aligned} \tan(A-B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \\ &= \frac{-4/5 - 12/5}{1 + (-4/5)(12/5)} \\ &= \frac{-16/5}{1 + -48/25} \\ &= \frac{-16/5}{-23/25} \\ &= \frac{16}{23} \end{aligned}$$

11) Sketch a graph of the equation  $y = 1 + 2 \sin 4x \cos 2x - 2 \cos 4x \sin 2x$ .

$$\begin{aligned} y &= 2(\sin 4x \cos 2x - \cos 4x \sin 2x) + 1 \\ &= 2 \sin(4x - 2x) + 1 \end{aligned}$$

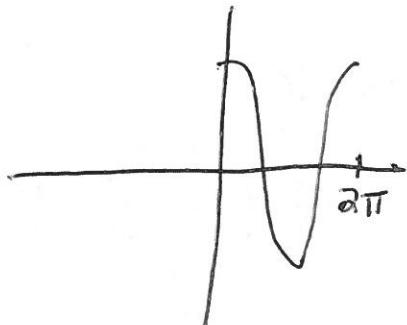
$$y = 2 \sin 2x + 1$$



12) Sketch a graph of the equation  $y = \cos^2(x/2) - \sin^2(x/2)$

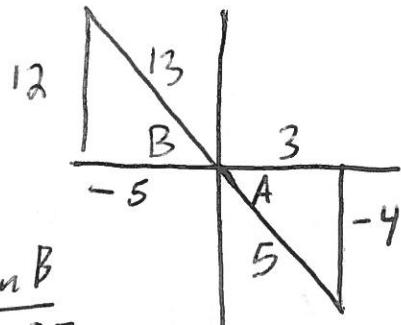
$$y = \cos(2 \cdot \frac{x}{2})$$

$$y = \cos x$$



① Draw Pics of A & B

② Use formula, flip at end, if reciprocal



d.  $\tan(2B)$

$$\frac{2 \tan B}{1 - \tan^2 B}$$

$$\begin{aligned} &2 \left(\frac{12}{5}\right) \\ &\frac{24}{1 - (\frac{12}{5})^2} \\ &\frac{24}{-119/25} \\ &= \frac{24}{-119/25} = \frac{120}{119} \end{aligned}$$

Use  $\sin(A-B)$  to

Condense. Then graph by finding.

(A),  $y = D$ , Period  $(\frac{2\pi}{B})$

&  $Bx = C$  for Starting point.

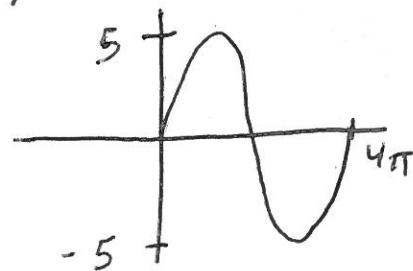
Use  $\cos 2A$  to condense.

13) Sketch a graph of the equation  $y = 10\sin(x/4)\cos(x/4)$

$$y = 5(2\sin \frac{x}{4} \cos \frac{x}{4})$$

$$y = 5 \sin \frac{2x}{4}$$

$$y = 5 \sin \frac{x}{2}$$



Use  $\sin 2A$

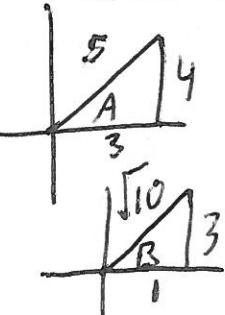
to condense.

14) Sketch a graph of the equation  $y = 1/(\cos(3x)\sin(2x)) - \cos(2x)\sin(3x)$

$$\begin{aligned} y &= \frac{1}{\sin(2x-3x)} \\ y &= \frac{1}{\sin(-x)} \\ y &= \csc(-x) \\ y &= -\csc(x) \quad \text{ODD Property.} \end{aligned}$$

Use  $\sin(A-B)$  to  
condense denominator.  
Rewrite  $\frac{1}{\sin x}$  as  
 $\csc x$  and  
graph.

15) Find the exact value of  $\cos(\arcsin(4/5) - \tan^{-1}(3))$



$$\begin{aligned} &\cos A \cos B + \sin A \sin B \\ &\left(\frac{3}{5}\right)\left(\frac{1}{\sqrt{10}}\right) + \left(\frac{4}{5}\right)\left(\frac{3}{\sqrt{10}}\right) \\ &\frac{15}{5\sqrt{10}} \cdot \frac{1}{\sqrt{10}} = \frac{15\sqrt{10}}{50} = \frac{3\sqrt{10}}{10} \end{aligned}$$

$$\begin{aligned} &\text{Let } A = \arcsin(4/5) \\ &B = \tan^{-1}(3) \end{aligned}$$

Draw pics of A & B.  
use  $\cos(A-B)$   
formula.

16) Evaluate the following.

$$2\tan(15)/\left(1 - \tan^2(15)\right)$$

$$\tan 2A$$

$$\frac{(\tan 140 - \tan 5)/(1 + \tan 5 \tan 140)}{A \quad B \quad B \quad A}$$

$$\tan(A-B)$$

$$\tan 2 \cdot 15$$

$$\tan 30 = \frac{\sqrt{3}}{3}$$

$$\tan(140-5)$$

$$\tan 135 = -1$$

$$\cos(\pi/5)\cos(19\pi/30) - \sin(\pi/5)\sin(19\pi/30)$$

$$\cos(36+116)$$

$$\cos 150$$

$$\left(-\frac{\sqrt{3}}{2}\right)$$

$$\cos(A+B)$$

$$\sec(75^\circ)$$

$$\cos(30+45)$$

$$= \cos 30 \cos 45 - \sin 30 \sin 45$$

$$\frac{\frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}}{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

use  $\cos\left(\frac{A}{2}\right)$

then flip.

$$\sec(75) = \frac{4}{\sqrt{6}-\sqrt{2}} \quad \text{or} \quad \frac{4}{\sqrt{6}+\sqrt{2}}$$

17) Evaluate the following.

$$\cos(105^\circ)$$

$$\cos\left(\frac{210}{2}\right) = -\sqrt{\frac{1+\cos 210}{2}} = -\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2+\sqrt{3}}{4}} = -\frac{\sqrt{2+\sqrt{3}}}{2}$$

$$\sin(195^\circ)$$

$$\text{use } \sin\left(\frac{A}{2}\right)$$

$$\sin\left(\frac{390}{2}\right) = +\sqrt{\frac{1-\cos 390}{2}} = +\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = +\sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

$$\sec(22.5^\circ)$$

$$\text{use } \cos\left(\frac{A}{2}\right), \text{ then flip.}$$

$$\cos\left(\frac{45}{2}\right) = +\sqrt{\frac{1+\cos 45}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\csc(67.5^\circ)$$

$$\text{use } \sin\left(\frac{A}{2}\right), \text{ then flip.}$$

$$\sin\left(\frac{135}{2}\right) = +\sqrt{\frac{1-\cos 135}{2}}$$

$$\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \frac{1-\frac{\sqrt{2}}{2}}{2}$$

$$\sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

$$\csc(67.5) = \frac{2}{\sqrt{2-\sqrt{2}}} \cdot \frac{\sqrt{2-\sqrt{2}}}{\sqrt{2-\sqrt{2}}} = \frac{2}{\sqrt{2-\sqrt{2}}}$$

$$\tan(255^\circ)$$

$$\tan(225+30)$$

$$\frac{\tan 225 + \tan 30}{1 - \tan 225 \tan 30} = \frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}} = \frac{(3 + \sqrt{3})(3 + \sqrt{3})}{(3 - \sqrt{3})(3 + \sqrt{3})} = \frac{9 + 6\sqrt{3} + 3}{9 - 3} = \frac{12 + 6\sqrt{3}}{6}$$

$$2 + \sqrt{3}$$

use  $\tan(A+B)$ , then flip.

$$\tan(105)$$

$$= \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45}$$

$$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \cdot \frac{(1+\sqrt{3})}{(1+\sqrt{3})} = \frac{1 + 2\sqrt{3} + 3}{-2} = \frac{4 + 2\sqrt{3}}{-2} = -(2 + \sqrt{3})$$

$$\cot(15^\circ)$$

$$\text{use } \tan(A-B)$$

~~$$\tan(45-30)$$~~

$$\frac{12-6\sqrt{3}}{6} = \frac{12-6\sqrt{3}}{1 + \tan 45 \tan 30} = \frac{12-6\sqrt{3}}{1 + \frac{\sqrt{3}}{3}} = \frac{12-6\sqrt{3}}{3 + \sqrt{3}}$$

$$\frac{9-6\sqrt{3}}{9-3} = \frac{9-6\sqrt{3}}{6} = \frac{3-\sqrt{3}}{2-\sqrt{3}} = \frac{3-\sqrt{3}}{(3+\sqrt{3})(3-\sqrt{3})} = \frac{3-\sqrt{3}}{6} = \frac{\sqrt{3}-1}{2}$$

$$\csc(345^\circ)$$

$$\text{use } \sin(A+B)$$

$$\sin(315+30)$$

$$\sin 315 \cos 30 + \cos 315 \sin 30 = \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) = \frac{\sqrt{2}-\sqrt{6}}{4}$$

$$-\frac{\sqrt{2}+\sqrt{6}}{2}$$

$$\frac{4(\sqrt{2}+\sqrt{6})}{4} = -4$$

$$\csc(345) = \frac{4(\sqrt{2}+\sqrt{6})}{\sqrt{2}-\sqrt{6}(\sqrt{2}+\sqrt{6})}$$

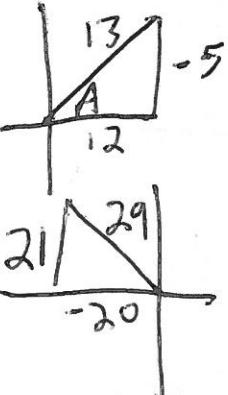
$$\boxed{2\sqrt{2}(2+\sqrt{2})}$$

$$\text{Let } A = \arcsin\left(-\frac{5}{13}\right) \text{ &}$$

$$B = \arccos\left(-\frac{20}{29}\right)$$

use  $\tan(A+B)$ .

18) Find the exact value of  $\tan(\arcsin(-5/13) + \arccos(-20/29))$



$$\begin{aligned} &\frac{\tan A + \tan B}{1 - \tan A \tan B} \\ &\frac{-\frac{5}{13} + \frac{21}{29}}{1 - \frac{5}{13} \cdot \frac{21}{29}} \end{aligned}$$

19). Given  $\cos A = -7/25$  in QII and  $\csc B = -13/12$  in QIV.

Find  $\cos(2A)$

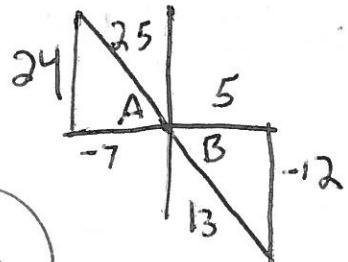
$$\begin{aligned} &\cos^2 A - \sin^2 A \\ &\left(\frac{-7}{25}\right)^2 - \left(\frac{24}{25}\right)^2 \\ &\frac{49}{625} - \frac{576}{625} \\ &\frac{-527}{625} \end{aligned}$$

Find  $\tan(A+B)$

$$\begin{aligned} &\frac{\tan A + \tan B}{1 - \tan A \tan B} \\ &\frac{\frac{24}{7} + \frac{-12}{5}}{1 - \frac{24}{7} \cdot \frac{-12}{5}} \\ &\frac{\frac{24}{7} + \frac{-84}{35}}{1 - \frac{-24 \cdot 12}{35}} \\ &\frac{\frac{35}{35} - \frac{288}{35}}{35 - 35} \rightarrow \frac{-253}{35} \end{aligned}$$

Draw pics of

A & B



20) Prove the identities.

$$\tan \frac{x}{2} + \cot \frac{x}{2} = 2 \csc x$$

$$\frac{1-\cos x}{\sin x} + \frac{\sin x}{1-\cos x} = \frac{1+\cos x}{1+\cos x}$$

$$\frac{1-\cos x}{\sin x} + \frac{\sin x(1+\cos x)}{\sin^2 x}$$

$$\frac{1-\cos x + 1+\cos x}{\sin x}$$

$$\frac{\sin x}{\frac{2}{\sin x}} = 2 \csc x \checkmark$$

$$\cos^2 \frac{x}{2} = \frac{\tan x + \sin x}{2 \tan x}$$

$$\sqrt{\frac{1+\cos A}{2}}^2$$

$$\frac{1+\cos A}{2}$$

$$\text{Use } \tan \frac{A}{2} = \frac{1-\cos A}{\sin A}$$

$$\text{or } \tan \frac{A}{2} = \frac{\sin A}{1+\cos A}$$

therefore

$$\cot \frac{A}{2} = \frac{\sin A}{1-\cos A} = \frac{1+\cos A}{\sin A}$$

$$\text{Use: } \cos \frac{A}{2}$$

$$\frac{\sin x}{\cos x + \sin x} \cdot \cos x$$

$$\frac{\sin x + \sin x \cos x}{2 \sin x}$$

$$\frac{1+\cos x}{2} \checkmark$$