The following examples use more than one of the rules at a time.

Example 4

Expand $\log_2\left(\frac{a^2b}{c}\right)$.

$\log_2\left(\frac{a^2b}{c}\right) = \log_2 a^2b - \log_2 c$ — Use the Quotient Rule for Logarithms.

$= \log_2 a^2 + \log_2 b - \log_2 c$ — Use the Product Rule for Logarithms.

$= 2 \cdot \log_2 a + \log_2 b - \log_2 c$ — Use the Power Rule for Logarithms

The answer is $2 \cdot \log_2 a + \log_2 b - \log_2 c$.

Example 5

Expand $\log_5 \sqrt{8a^7}$.

$\log_5 \sqrt{8a^7} = \log_5 (8a^7)^{1/2}$ — Rewrite the radical with a fractional exponent.

$= \frac{1}{2}\log_5 (8a^7)$ — Use the Power Rule for Logarithms.

$= \frac{1}{2}(\log_5 8 + \log a^7)$ — Use the Product Rule for Logarithms.

$= \frac{1}{2}(\log_5 8 + 7\log a)$ — Use the Power Rule for Logarithms.

The answer is $\frac{1}{2}(\log_5 8 + 7\log a)$

II) Exercises

Expand the following logarithms.
Use either the power rule, product rule or quotient rule.

1. $\log_2(9^5)$ = _____

2. $\log_2(21)$ = _____

3. $\log_5\left(\frac{19}{2}\right)$ = _____

4. $\log_2(6a)$ = _____

5. $\log_3(xy)$ = _____

6. $\log_5\left(\frac{a}{3}\right)$ = _____

7. $\log_3(5y)$ = _____

8. $\log_3(a^{10})$ = _____

Expand the following logarithms using one or more of the logarithm rules.

9. $\log_5\left(\frac{12a}{2}\right)$ = _____

10. $\log_2\left(\frac{a}{b}\right)^5$ = _____

11. $\log_5\sqrt{x^5y}$ = _____

12. $\log_5\left(\frac{xy}{z}\right)^8$ = _____

13. $\log_2\left(\frac{1-x}{y}\right)^3$ = _____

14. $\log_3\sqrt[5]{9x^3}$ = _____

15. $\log_3\sqrt[3]{2x^5}$ = _____

16. $\log_2\left(\frac{9x^{10}}{y^2}\right)$ = _____

17. $\log_2\left(\frac{4a}{5}\right)$ = _____

18. $\log_2\sqrt[3]{x^2a}$ = _____

Sometimes you need to write an expression as a single logarithm.
Use the rules to work backwards.

Example 6

Write $2\log_3 x + \log_3 y$ as a single logarithm

$\log_3 x^2 + \log_3 y$	Use the Power Rule for Logarithms to move the 2 in $2\log_3 x$ to the exponent of x
$= \log_3 x^2y$	Use the Product Rule for Logarithms.

The answer is $\log_3 x^2y$

Example 7

Simplify $\frac{1}{2}\log_5 100 - \log_5 2$

$\log_5 100^{1/2} - \log_5 2$	Use the Power Rule for Logarithms.
$= \log_5 10 - \log_5 2$	Simplify.
$= \log_5(10 \div 2) = \log_5 5$	Use the Quotient Rule for Logarithms.
$= 1$	Simplify.

The answer is 1

III) Rewrite as Single Expression
Write as a single logarithm.

19. $2\log_3 10 - \log_3 4 =$ _____

20. $\frac{2}{3}\log_2 x + \log_2 y =$ _____

21. $\frac{1}{2}\log_5 x + \log_5 y =$ _____

22. $3\log_3 x + 4\log_3 y =$ _____

23. $6\log_3 x + 2\log_3 11 =$ _____

24. $4\log_5 x - \log_5 y + \log_5 z =$ _____

25. $\frac{1}{2}\log_3 144 - \log_3 4 =$ _____

26. $\log_3 a + \log_3 b - 2\log_3 c =$ _____

IV) Extension Problems

27. Let $\log_b 2 = x$, $\log_b 3 = y$ and $\log_b 5 = z$.

(a) What is the value of $\log_b 50$ in terms of x, y and z?

(b) What is the value of $\log_b 3000$ in terms of x, y and z?

28. Are $\log_2 16$ and $\log_4 64$ equal? Why or why not?

29. Correct the error
There is an error in the student work shown below.

Directions: Simplify $\log_2 (6x)^5$.

$$\log_2 (6x)^5 = 5 \bullet \log_2 (6 \bullet x)$$
$$= 5 \bullet \log_2 6 + \log_2 x$$
$$= 5\log_2 6 + \log_2 x$$

What is the error in the work above?

_____

_____

Answer Key

1. $5 \log_2 9 = 10 \log_2 3$
2. $\log_2 3 + \log_2 7$
3. $\log_5 19 - \log_5 2$
4. $\log_2 6 + \log_2 a$
5. $\log_3 x + \log_3 y$
6. $\log_5 a - \log_5 3$
7. $\log_3 5 + \log_3 y$
8. $10 \log_3 a$
9. $\log_5 6 + \log_5 a$
10. $5 (\log_2 a - \log_2 b)$
11. $\frac{1}{2}(5\log_5 x + \log_5 y)$
12. $8 (\log_5 x + \log_5 y - \log_5 z)$
13. $3 (\log_2 (1 - x) - \log_2 y)$
14. $\frac{1}{5}(2 - 3\log_3 x)$
15. $\frac{1}{3}(\log_3 2 - 5\log_3 x)$
16. $2 \log_2 3 + 10 \log_2 x - 2 \log_2 y$
17. $2 + \log_2 a - \log_2 5$
18. $\frac{1}{3}(2\log_2 x + \log_2 a)$
19. $\log_3 25$
20. $\log_2 (x^{2/3} y)$
21. $\log_5 (x^{1/2} y)$
22. $\log_3 (x^3 y^4)$
23. $\log_3 (121 x^6)$
24. $\log_5 \left(\frac{x^4 z}{y} \right)$
25. 1
26. $\log_5 \left(\frac{ab}{c^2} \right)$
27. (a) $x + y + z$; (b) $3(x + z) + y$
28. Yes; they are both equal to 4.
29. The student did not distribute the 5 to $\log_2 6$ and $\log_2 x$; the correct answer is $5(\log_2 6 + \log_2 x)$, or $5 \log_2 6 + 5 \log_2 x$.