Therefore,

$$\begin{aligned}
\tan(A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\
&= \frac{\dfrac{3}{4} + \dfrac{12}{5}}{1 - \dfrac{3}{4} \cdot \dfrac{12}{5}} \\
&= \frac{\dfrac{15}{20} + \dfrac{48}{20}}{1 - \dfrac{9}{5}} \\
&= \frac{\dfrac{63}{20}}{-\dfrac{4}{5}} \\
&= -\frac{63}{16}
\end{aligned}$$

which is the same result we obtained previously.

GETTING READY FOR CLASS

After reading through the preceding section, respond in your own words a in complete sentences.

a. Why is it necessary to have sum and difference formulas for sine, cosi and tangent?

b. Write both the sum and the difference formulas for cosine.

c. Write both the sum and the difference formulas for sine.

d. Write both the sum and the difference formulas for tangent.

PROBLEM SET 5.2

Find exact values for each of the following:

1. $\sin 15°$

2. $\sin 75°$

3. $\tan 15°$

4. $\tan 75°$

5. $\sin \dfrac{7\pi}{12}$

6. $\cos \dfrac{7\pi}{12}$

7. $\cos 105°$

8. $\sin 105°$

Show that each of the following is true:

9. $\sin(x + 2\pi) = \sin x$

10. $\cos(x - 2\pi) = \cos x$

11. $\cos\left(x - \dfrac{\pi}{2}\right) = \sin x$

12. $\sin\left(x - \dfrac{\pi}{2}\right) = -\cos x$

13. $\cos(180° - \theta) = -\cos\theta$

14. $\sin(180° - \theta) = \sin\theta$

15. $\sin(90° + \theta) = \cos\theta$

16. $\cos(90° + \theta) = -\sin\theta$

17. $\tan\left(x + \dfrac{\pi}{4}\right) = \dfrac{1 + \tan x}{1 - \tan x}$

18. $\tan\left(x - \dfrac{\pi}{4}\right) = \dfrac{\tan x - 1}{\tan x + 1}$

19. $\sin\left(\dfrac{3\pi}{2} - x\right) = -\cos x$

20. $\cos\left(x - \dfrac{3\pi}{2}\right) = -\sin x$

Write each expression as a single trigonometric function.

21. $\sin 3x \cos 2x + \cos 3x \sin 2x$

22. $\cos 3x \cos 2x + \sin 3x \sin 2x$

23. $\cos 5x \cos x - \sin 5x \sin x$

24. $\sin 8x \cos x - \cos 8x \sin x$

25. $\cos 15° \cos 75° - \sin 15° \sin 75°$

26. $\cos 15° \cos 75° + \sin 15° \sin 75°$

Graph each of the following from $x = 0$ to $x = 2\pi$.

27. $y = \sin 5x \cos 3x - \cos 5x \sin 3x$

28. $y = \sin x \cos 2x + \cos x \sin 2x$

29. $y = 3 \cos 7x \cos 5x + 3 \sin 7x \sin 5x$

30. $y = 2 \cos 4x \cos x + 2 \sin 4x \sin x$

31. Graph one complete cycle of $y = \sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}$ by first rewriting the right side in the form $\sin(A + B)$.

32. Graph one complete cycle of $y = \sin x \cos \frac{\pi}{6} - \cos x \sin \frac{\pi}{6}$ by first rewriting the right side in the form $\sin(A - B)$.

33. Graph one complete cycle of $y = 2(\sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3})$ by first rewriting the right side in the form $2\sin(A + B)$.

34. Graph one complete cycle of $y = 2(\sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3})$ by first rewriting the right side in the form $2\sin(A - B)$.

35. Let $\sin A = \frac{3}{5}$ with A in QII and $\sin B = -\frac{5}{13}$ with B in QIII. Find $\sin(A + B)$, $\cos(A + B)$, and $\tan(A + B)$. In what quadrant does $A + B$ terminate?

36. Let $\cos A = -\frac{5}{13}$ with A in QII and $\sin B = \frac{3}{5}$ with B in QI. Find $\sin(A - B)$, $\cos(A - B)$, and $\tan(A - B)$. In what quadrant does $A - B$ terminate?

37. If $\sin A = 1/\sqrt{5}$ with A in QI and $\tan B = \frac{3}{4}$ with B in QI, find $\tan(A + B)$ and $\cot(A + B)$. In what quadrant does $A + B$ terminate?

38. If $\sec A = \sqrt{5}$ with A in QI and $\sec B = \sqrt{10}$ with B in QI, find $\sec(A + B)$. [First find $\cos(A + B)$.]

39. If $\tan(A + B) = 3$ and $\tan B = \frac{1}{2}$, find $\tan A$.

40. If $\tan(A + B) = 2$ and $\tan B = \frac{1}{3}$, find $\tan A$.

41. Write a formula for $\sin 2x$ by writing $\sin 2x$ as $\sin(x + x)$ and using the formula for the sine of a sum.

42. Write a formula for $\cos 2x$ by writing $\cos 2x$ as $\cos(x + x)$ and using the formula for the cosine of a sum.

Prove each identity.

43. $\sin(90° + x) + \sin(90° - x) = 2\cos x$

44. $\sin(90° + x) - \sin(90° - x) = 0$

45. $\cos(x - 90°) - \cos(x + 90°) = 2\sin x$

46. $\cos(x + 90°) + \cos(x - 90°) = 0$