

Name: _____ Period: _____ Date: _____

Sum, Difference and Double Angle Formulas for Sine and Cosine

Directions: Answer these questions without a calculator.

1. Find the exact value for $\cos(165^\circ)$.

$$\begin{aligned} & \cos(120 + 45) \\ &= \cos 120 \cos 45 - \sin 120 \sin 45 \\ & \quad \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ & \quad \frac{-\sqrt{2} - \sqrt{6}}{4} \end{aligned}$$

2. Find the exact value for $\sin\left(\frac{7\pi}{12}\right) = \sin 105$

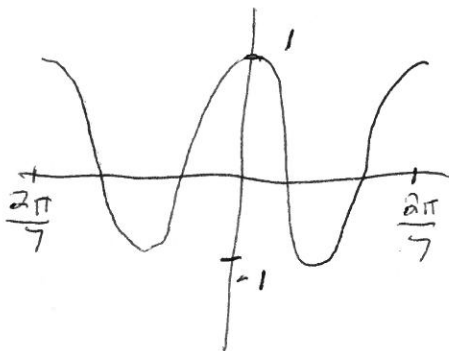
$$\begin{aligned} & \sin(60 + 45) \\ &= \sin 60 \cos 45 + \cos 60 \sin 45 \\ & \quad \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \\ & \quad \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

3. Show that $\cos(x + 2\pi) = \cos(x)$

$$\begin{array}{l|l} \cos x \cos 2\pi - \sin x \sin 2\pi & \downarrow \\ \cos x(1) - \sin x(0) & \\ \hline \cos x & \cos x \end{array}$$

4. Sketch a graph $\cos(6x)\cos(x) - \sin(6x)\sin(x)$.

$$y = \cos(7x)$$

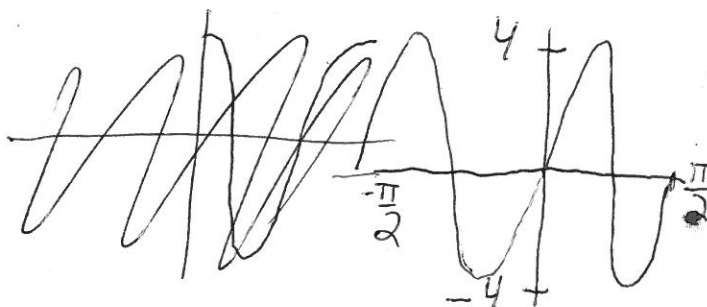


5. Show that $\cos(90^\circ - A) = \sin A$

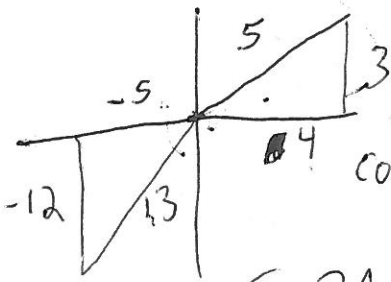
$$\begin{array}{l|l} \cos 90 \cos A + \sin 90 \sin A & \downarrow \\ 0 \cdot \cos A + 1 \cdot \sin A & \\ \hline \sin A & \sin A \end{array}$$

6. Graph $y = 4\sin 7x \cos 3x - 4\cos 7x \sin 3x$ over one full period.

$$\begin{aligned} y &= 4(\sin 7x \cos 3x - \cos 7x \sin 3x) \\ y &= 4 \sin(7x - 3x) \\ y &= 4 \sin 4x \end{aligned}$$



7. If $\sin A = 3/5$ in Q1 and $\cos B = -5/13$ with B in QIII, find $\sin(A+B)$, $\cos(A-B)$, $\sin(2A)$, $\cos(2A)$.



$$\sin(A+B) = \sin A \cos B + \cos A \sin B = \left(\frac{3}{5}\right)\left(\frac{-5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{-12}{13}\right) = \frac{-15-48}{65} = \frac{-63}{65}$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B = \left(\frac{4}{5}\right)\left(\frac{-5}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{-12}{13}\right) = \frac{-20-36}{65} = \frac{-56}{65}$$

$$\sin 2A = 2 \sin A \cos A = 2 \left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$$

$$\cos 2A = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

8. Find an exact value for $\cos 15^\circ \cos 75^\circ - \sin 15^\circ \sin 75^\circ$.

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(15+75)$$

$$\cos 90$$

0

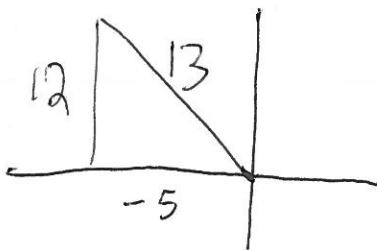
9. Find an exact value for $\sin 68^\circ \cos 23^\circ - \cos 68^\circ \sin 23^\circ$.

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\sin(68-23) = \sin 45$$

$$\frac{\sqrt{2}}{2}$$

10. If $\tan A = -12/5$ in Q2 and find $\sin(2A)$ and $\cos(2A)$.



$$\sin 2A = 2 \sin A \cos A$$

$$2 \left(\frac{12}{13}\right)\left(\frac{-5}{13}\right)$$

$$\frac{-120}{169}$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$2 \left(\frac{-5}{13}\right)^2 - 1$$

$$2 \cdot \frac{25}{169} - 1$$

$$\frac{50}{169} - \frac{169}{169}$$

$$-\frac{119}{169}$$

11. Prove $\frac{2-2\cos 2x}{\sin 2x} = \sec x (\csc x) - \cot x + \tan x$

12. Prove $(\cos x - \sin x)(\cos x + \sin x) = \cos 2x$.

Try these, we will go over them on Monday.