Graphing to Figure Some Formulas

To do this activity you’re going to need approximations for \( \frac{2\pi}{n} \), where \( n \) ranges from 1 to 6. Find them now and jot them down in the space provided.

<table>
<thead>
<tr>
<th>( n )</th>
<th>Exact</th>
<th>Approximate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
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<tr>
<td>3</td>
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<tr>
<td>4</td>
<td></td>
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<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

On your calculator graph the following function:

\[
y = \sin(2x)\cos(x) + \cos(2x)\sin(x)
\]

Does the graph look familiar despite how unfamiliar the equation looks?

Sketch the graph in the space below:

Use the intersection or zero function built into your calculator to determine the approximate period of the graph you’re looking at. Look up that number in the table you filled in above and find the exact period.

\[
\text{Period} = \quad \Rightarrow \quad B = \quad
\]

Use trace to quickly approximate the maximum and minimum values of the graph you’re looking at. The actual values are nice numbers. What are the maximum and minimum values?

\[
\begin{align*}
\text{Max} &= \quad \\
\text{min} &= \quad
\end{align*}
\quad \Rightarrow 
\begin{align*}
\text{Amp} &= \quad \\
\text{S.A.} &= \quad
\end{align*}
\]

Write a new function for the graph you are looking at.

If two functions have the exact same graph, then they are equivalent. Therefore,

\[
\sin(2x)\cos(x) + \cos(2x)\sin(x) = \quad
\]
Repeat this process on each of the following functions. This shouldn’t take all that long so work quickly.

1. \(y = \sin(5x)\cos(x) + \cos(5x)\sin(x) = \) __________

2. \(y = \sin(2x)\cos(4x) + \cos(2x)\sin(4x) = \) __________

3. \(y = \sin(4x)\cos(x) + \cos(4x)\sin(x) = \) __________

4. \(y = \sin(2x)\cos(3x) + \cos(2x)\sin(3x) = \) __________

5. \(y = \sin(x)\cos(3x) + \cos(x)\sin(3x) = \) __________

6. \(y = \sin(2x)\cos(2x) + \cos(2x)\sin(2x) = \) __________

Look carefully at the original equations and the equations you came up with…does some general thing seem to be happening here?

**Sum formula for Sine**

\[
\sin(A)\cos(B) + \cos(A)\sin(B) = __________
\]

There’s actually a special case of this formula. What happens if \(A = B\)? We can do this algebraically, which is a bit quicker than doing the whole graphing thing.

<table>
<thead>
<tr>
<th>Step 1: Replace all the (B)’s with (A)’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2: Simplify as much as possible.</td>
</tr>
</tbody>
</table>

The formula we just found is called a *double angle formula.*

**Double Angle Formula for Sine**

\[
\text{Graph } y = 2\sin\left(\frac{x}{4}\right)\cos\left(\frac{x}{4}\right) \text{ in the space below.}
\]