LCD

Add numerators

Expand $(1 + \cos \alpha)^2$

Pythagorean identity

Factor out a 2

Reduce

Reciprocal identity

it we multiply the numera-
s is similar to rationalizing

numerator and
inator by $1 + \sin t$

out the denominator

ean identity

this identity by multiplying
sin t.

in your own words and

$\frac{1 - \sin^4 t}{\cos^2 t}$?

$+ \frac{1 + \cos \alpha}{\sin \alpha}$?

PROBLEM SET 5.1

Prove that each of the following identities is true:

1. $\cos \theta \tan \theta = \sin \theta$
2. $\sec \theta \cot \theta = \csc \theta$
3. $\csc \theta \tan \theta = \sec \theta$
4. $\tan \theta \cot \theta = 1$
5. $\dfrac{\tan A}{\sec A} = \sin A$
6. $\dfrac{\cot A}{\csc A} = \cos A$
7. $\sec \theta \cot \theta \sin \theta = 1$
8. $\tan \theta \csc \theta \cos \theta = 1$
9. $\cos x (\csc x + \tan x) = \cot x + \sin x$
10. $\sin x (\sec x + \csc x) = \tan x + 1$
11. $\cot x - 1 = \cos x (\csc x - \sec x)$
12. $\tan x (\cos x + \cot x) = \sin x + 1$
13. $\cos^2 x (1 + \tan^2 x) = 1$
14. $\sin^2 x (\cot^2 x + 1) = 1$
15. $(1 - \sin x)(1 + \sin x) = \cos^2 x$
16. $(1 - \cos x)(1 + \cos x) = \sin^2 x$
17. $\dfrac{\cos^4 t - \sin^4 t}{\sin^2 t} = \cot^2 t - 1$
18. $\dfrac{\sin^4 t - \cos^4 t}{\sin^2 t \cos^2 t} = \sec^2 t - \csc^2 t$
19. $1 + \sin \theta = \dfrac{\cos^2 \theta}{1 - \sin \theta}$
20. $1 - \sin \theta = \dfrac{\cos^2 \theta}{1 + \sin \theta}$
21. $\dfrac{1 - \sin^4 \theta}{1 + \sin^2 \theta} = \cos^2 \theta$
22. $\dfrac{1 - \cos^4 \theta}{1 + \cos^2 \theta} = \sin^2 \theta$
23. $\sec^2 \theta - \tan^2 \theta = 1$
24. $\csc^2 \theta - \cot^2 \theta = 1$
25. $\sec^4 \theta - \tan^4 \theta = \dfrac{1 + \sin^2 \theta}{\cos^2 \theta}$
26. $\csc^4 \theta - \cot^4 \theta = \dfrac{1 + \cos^2 \theta}{\sin^2 \theta}$
27. $\tan \theta - \cot \theta = \dfrac{\sin^2 \theta - \cos^2 \theta}{\sin \theta \cos \theta}$
28. $\sec \theta - \csc \theta = \dfrac{\sin \theta - \cos \theta}{\sin \theta \cos \theta}$
29. $\csc B - \sin B = \cot B \cos B$
30. $\sec B - \cos B = \tan B \sin B$
31. $\cot \theta \cos \theta + \sin \theta = \csc \theta$
32. $\tan \theta \sin \theta + \cos \theta = \sec \theta$
33. $\dfrac{\cos x}{1 + \sin x} + \dfrac{1 + \sin x}{\cos x} = 2 \sec x$
34. $\dfrac{\cos x}{1 + \sin x} - \dfrac{1 - \sin x}{\cos x} = 0$
35. $\dfrac{1}{1 + \cos x} + \dfrac{1}{1 - \cos x} = 2 \csc^2 x$
36. $\dfrac{1}{1 - \sin x} + \dfrac{1}{1 + \sin x} = 2 \sec^2 x$
37. $\dfrac{1 - \sec x}{1 + \sec x} = \dfrac{\cos x - 1}{\cos x + 1}$
38. $\dfrac{\csc x - 1}{\csc x + 1} = \dfrac{1 - \sin x}{1 + \sin x}$
39. $\dfrac{\cos t}{1 + \sin t} = \dfrac{1 - \sin t}{\cos t}$
40. $\dfrac{\sin t}{1 + \cos t} = \dfrac{1 - \cos t}{\sin t}$
41. $\dfrac{(1 - \sin t)^2}{\cos^2 t} = \dfrac{1 - \sin t}{1 + \sin t}$
42. $\dfrac{\sin^2 t}{(1 - \cos t)^2} = \dfrac{1 + \cos t}{1 - \cos t}$
43. $\dfrac{\sec \theta + 1}{\tan \theta} = \dfrac{\tan \theta}{\sec \theta - 1}$
44. $\dfrac{\csc \theta - 1}{\cot \theta} = \dfrac{\cot \theta}{\csc \theta + 1}$
45. $\dfrac{1 - \sin x}{1 + \sin x} = (\sec x - \tan x)^2$
46. $\dfrac{1 + \cos x}{1 - \cos x} = (\csc x + \cot x)^2$
47. $\sec x + \tan x = \dfrac{1}{\sec x - \tan x}$
48. $\dfrac{1}{\csc x - \cot x} = \csc x + \cot x$
49. $\dfrac{\sin x + 1}{\cos x + \cot x} = \tan x$
50. $\dfrac{\cos x + 1}{\cot x} = \sin x + \tan x$
51. $\sin^4 A - \cos^4 A = 1 - 2 \cos^2 A$
52. $\cos^4 A - \sin^4 A = 1 - 2 \sin^2 A$

53. $\dfrac{\sin^2 B - \tan^2 B}{1 - \sec^2 B} = \sin^2 B$

54. $\dfrac{\cot^2 B - \cos^2 B}{\csc^2 B - 1} = \cos^2 B$

55. $\dfrac{\sec^4 y - \tan^4 y}{\sec^2 y + \tan^2 y} = 1$

56. $\dfrac{\csc^2 y + \cot^2 y}{\csc^4 y - \cot^4 y} = 1$

57. $\dfrac{\sin^3 A - 8}{\sin A - 2} = \sin^2 A + 2 \sin A + 4$

58. $\dfrac{1 - \cos^3 A}{1 - \cos A} = \cos^2 A + \cos A + 1$

59. $\dfrac{1 - \tan^3 t}{1 - \tan t} = \sec^2 t + \tan t$

60. $\dfrac{1 + \cot^3 t}{1 + \cot t} = \csc^2 t - \cot t$

61. $\dfrac{\tan x}{\sin x - \cos x} = \dfrac{\sin^2 x + \sin x \cos x}{\cos x - 2 \cos^3 x}$

62. $\dfrac{\cot^2 x}{\sin x + \cos x} = \dfrac{\cos^2 x \sin x - \cos^3 x}{2 \sin^4 x - \sin^2 x}$

The following identities are from the book *Plane and Spherical Trigonometry with Tables* by Rosenbach, Whitman, and Moskovitz, and published by Ginn and Company in 1937. Verify each identity.

63. $(\tan \theta + \cot \theta)^2 = \sec^2 \theta + \csc^2 \theta$

64. $\dfrac{\tan^2 \psi + 2}{1 + \tan^2 \psi} = 1 + \cos^2 \psi$

65. $\dfrac{1 + \sin \phi}{1 - \sin \phi} - \dfrac{1 - \sin \phi}{1 + \sin \phi} = 4 \tan \phi \sec \phi$

66. $\dfrac{\cos \beta}{1 - \tan \beta} + \dfrac{\sin \beta}{1 - \cot \beta} = \sin \beta + \cos \beta$

Use your graphing calculator to determine if each equation appears to be an identity or not by graphing the left expression and right expression together.

67. $(\sec B - 1)(\sec B + 1) = \tan^2 B$

68. $\dfrac{1 - \sec \theta}{\cos \theta} = \dfrac{\cos \theta}{1 + \sec \theta}$

69. $\sec x + \cos x = \tan x \sin x$

70. $\dfrac{\tan t}{\sec t + 1} = \dfrac{\sec t - 1}{\tan t}$

71. $\sec A - \csc A = \dfrac{\cos A - \sin A}{\cos A \sin A}$

72. $\cos^4 \theta - \sin^4 \theta = 2 \cos^2 \theta - 1$

73. $\dfrac{1}{1 - \sin x} + \dfrac{1}{1 + \sin x} = 2 \sec^2 x$

74. $\cot^4 t - \tan^4 t = \dfrac{\sin^2 t + 1}{\cos^2 t}$

Show that each of the following statements is not an identity by finding a value of θ that makes the statement false.

75. $\sin \theta = \sqrt{1 - \cos^2 \theta}$

76. $\sin \theta + \cos \theta = 1$

77. $\sin \theta = \dfrac{1}{\cos \theta}$

78. $\tan^2 \theta + \cot^2 \theta = 1$

79. $\sqrt{\sin^2 \theta + \cos^2 \theta} = \sin \theta + \cos \theta$

80. $\sin \theta \cos \theta = 1$