Name: AK

Algebra II Honors

For each problem: a) list all possible rational zeros

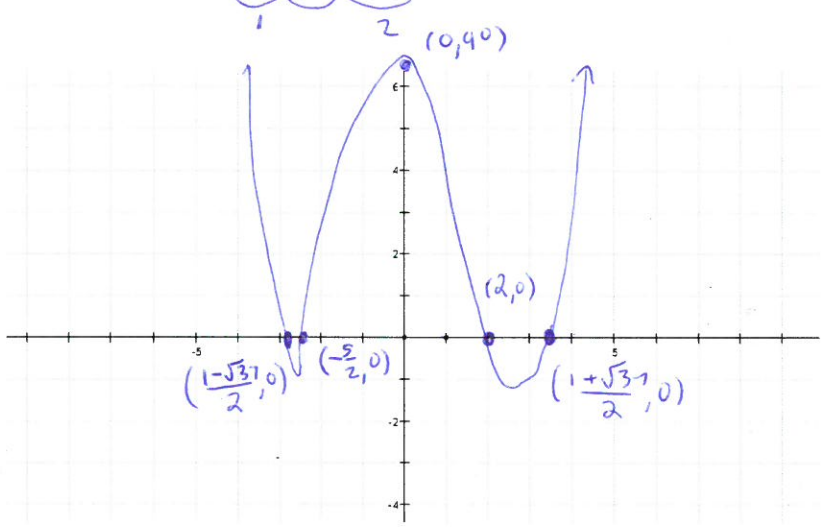
b) apply Descartes's Rule to understand more about the zeros

c) find **all** zeros by using synthetic or long division (one or two may be given to you)

d) factor over the rational numbers

e) sketch the polynomial, identifying all intercepts

1. $f(x) = 2x^4 - x^3 - 29x^2 + x + 90$ (zeros: 2, -5/2)



a) $\frac{\pm 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90}{1, 2}$

b)

Pos real	Neg real	Imaginary
2	2	0
[redacted]	[redacted]	[redacted]
0	2	2
2	0	2
[redacted]		
0	0	4

$F(-x) = 2x^4 + x^3 - 29x^2 - x + 90$

c)

2	2	-1	-29	1	90
	↓	4	6	-46	-90
-5/2	2	3	-23	-45	0
	↓	-5	5	45	
	2	-2	-18	0	

d) $(x-2)(x+5/2)(2x^2-2x-18)$

$(x-2)(x+5/2)2(x^2-x-9)$

[redacted]

$\frac{1 \pm \sqrt{1-4(-9)}}{2}$

$\frac{1 \pm \sqrt{37}}{2}$

$(x-2)(x+5/2)(2)\left(x-\left(\frac{1+\sqrt{37}}{2}\right)\right)\left(x-\left(\frac{1-\sqrt{37}}{2}\right)\right)$

Factored completely

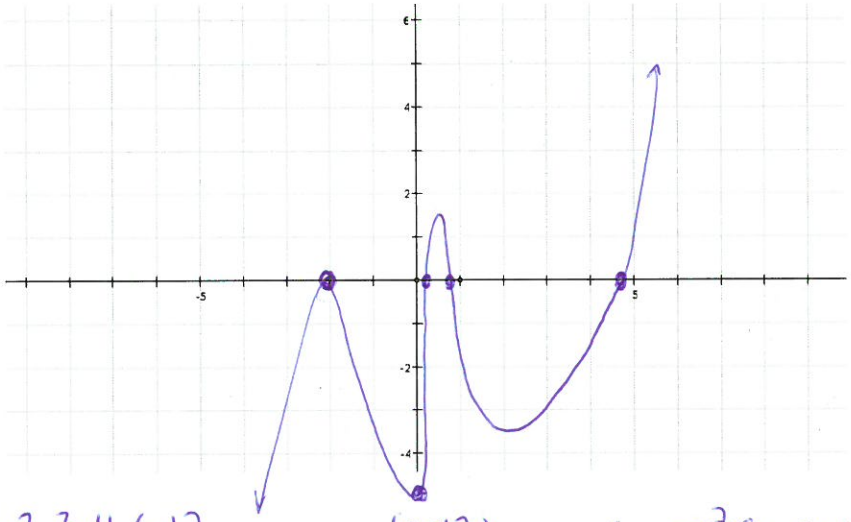
Factored over rationals

$(x-2)(2x+5)(x^2-x-9)$

2. $f(x) = 4x^5 - 7x^4 - 57x^3 - 19x^2 + 64x - 12$ (zeros: ¾, -2)

* hint: this has a double zero

1 2 3



a) $\pm \dfrac{1, 2, 3, 4, 6, 12}{1, 2, 4}$ $(0, -12)$

$(x+2)^2(x - 3/4)(4x^2 - 20x + 4)$

$(x+2)^2(x - 3/4)\,4(x^2 - 5x + 1)$

$(x+2)^2(4x - 3)(x^2 - 5x + 1)$

Factored over rationals

$x = \dfrac{5 \pm \sqrt{25 - 4(1)}}{2}$

$\dfrac{5 \pm \sqrt{21}}{2}$ → 4.8, → 0.2

b)

Pos real zeros	Neg Real zeros	Imaginary Zeros
3	2	0
3	0	2
1	2	2
1	0	4

$F(-x) = -4x^5 - 7x^4 + 57x^3 - 19x^2 - 64x - 12$

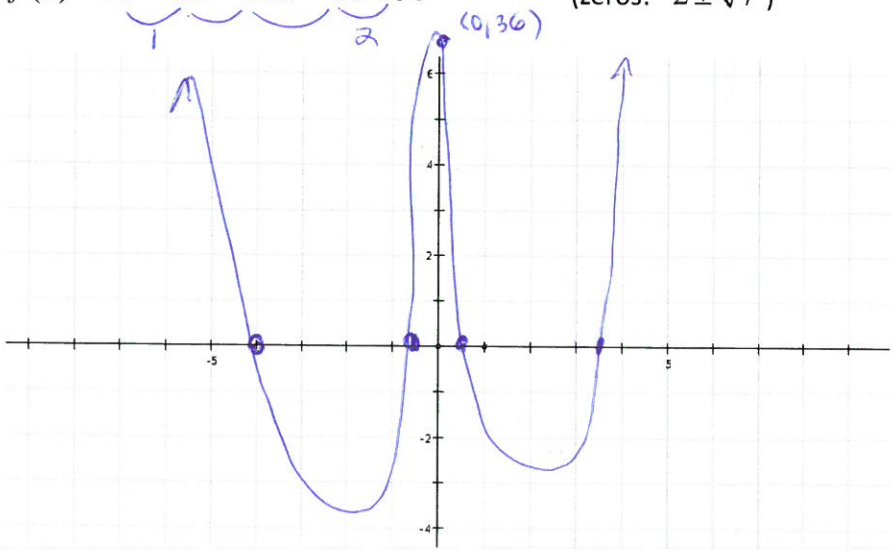
1 2

c)

-2	4	-7	-57	-19	64	-12
	↓	-8	30	54	-70	12
3/4	4	-15	-27	35	-6	0
	↓	3	-9	-27	6	
-2	4	-12	-36	8	0	
	↓	-8	40	-8		
	4	-20	4	0		

2 ± 2.5 / $-.5$
4.5

3. $f(x) = 5x^4 - 3x^3 - 95x^2 - 3x + 36$ (zeros: $2 \pm \sqrt{7}$)



-60
$1 \quad -60$
$2 \quad -30$
$3 \quad -20$
$-3 \quad 20$

a) $\pm \dfrac{1, 2, 3, 4, 6, 9, 12, 18, 36}{1, 5}$

b)

Pos real	Neg real	Imaginary
2	2	0
2	0	2
0	2	2
0	0	4

$F(-x) = 5x^4 + 3x^3 - 95x^2 + 3x + 36$
1 2

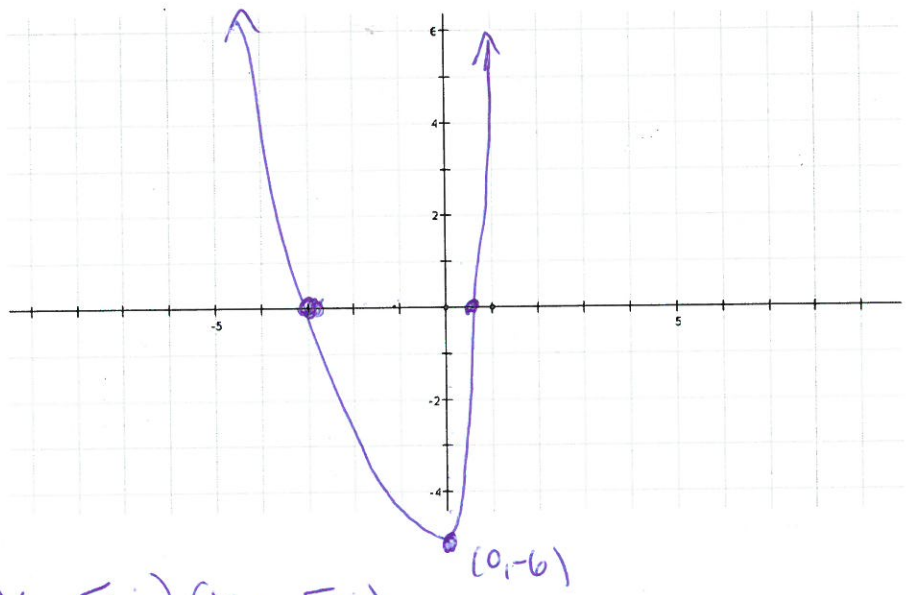
c) $(x - (2+\sqrt{7}))(x - (2-\sqrt{7}))$
$((x-2) - \sqrt{7})((x-2) + \sqrt{7})$
$(x-2)^2 - 7$
$x^2 - 4x + 4 - 7$

$$
\begin{array}{r}
5x^2 + 17x - 12 \\
x^2 - 4x - 3 \overline{\big) 5x^4 - 3x^3 - 95x^2 - 3x + 36} \\
-(5x^4 - 20x^3 - 15x^2) \\
\hline
17x^3 - 80x^2 - 3x \\
-(17x^3 - 68x^2 - 51x) \\
\hline
-12x^2 + 48x + 36 \\
-12x^2 + 48x + 36 \\
\hline
0
\end{array}
$$

$(x^2 - 4x - 3)(5x^2 + 17x - 12)$
$5x^2 + 20x - 3x - 12$
$5x(x+4) - 3(x+4)$
$(x^2 - 4x - 3)(x+4)(5x-3)$
Factored over rationals.

4. $f(x) = 2x^4 + 5x^3 + x^2 + 10x - 6$ (zero: $\sqrt{2}i$)

e)



c) $(x - \sqrt{2}i)(x + \sqrt{2}i)$

$$
\begin{array}{r|l}
 & 2x^2 + 5x - 3 \\
\hline
x^2 + 2 & 2x^4 + 5x^3 + x^2 + 10x - 6 \\
 & 2x^4 \qquad\quad 4x^2 \\
\hline
 & 5x^3 - 3x^2 + 10x - 6 \\
 & 5x^3 \qquad\quad +10x \\
\hline
 & -3x^2 \quad -6 \\
 & -3x^2 \quad -6 \\
\hline
 & 0
\end{array}
$$

$(x^2+2)(2x^2+5x-3)$

d) $(x^2+2)(2x-1)(x+3)$

$F(-x) = 2x^4 - 5x^3 + x^2 - 10x - 6$

1 2 3

a) $\frac{\pm 1, 2, 3, 6}{1, 2}$

b)

Pos real	Neg real	Imaginary
1	3	0
1	1	2