

Arc Length

We've already defined radians and talked about a formula for how to calculate them.

$$\theta = \frac{s}{r} = \frac{\text{arc length}}{\text{radius}}$$

From this it's not a huge leap to find a formula that will give us the arc length if we know the radius and the central angle (in radians!)...find that formula!

Arc Length:

If θ is a central angle in a circle with radius r , then the length of the arc cut off by θ is given by

$s = r\theta$, θ in radians.

Using the formula is pretty straightforward. Solve for whatever is missing in the table below.

s	r	θ
	25	$\frac{5\pi}{3}$
24π	12	
$\frac{28\pi}{5}$		$\frac{7\pi}{8}$

An assortment of clock problems...

Suppose that the minute hand of a clock is 2.5 inches long. Find the length of the arc traced out by the minute hand from 12:10 to 12:35.

Suppose that the minute hand of a clock is 4 feet long. Find the length of the arc traced out by the minute hand from 12:30 to 1:10.

Big Ben, in England, has an hour hand that is 9 feet long and a minute hand that is 14.3 feet long. In any 45 minute period how much farther does the tip of the minute hand move than the tip of the hour hand? Through what angle does each hand pass in that time period? (According to wikipedia—so you know it's true—Big Ben is actually the nickname of the bell inside the clock and the whole building is more properly known as the Clock Tower.)

The earth has a radius of approximately 6378 km. Find the distance between two cities that are directly north of each other if one city has latitude $19^{\circ}25'38''$ and the other has latitude $36^{\circ}14'27''$. Sketch a figure to help figure this out.

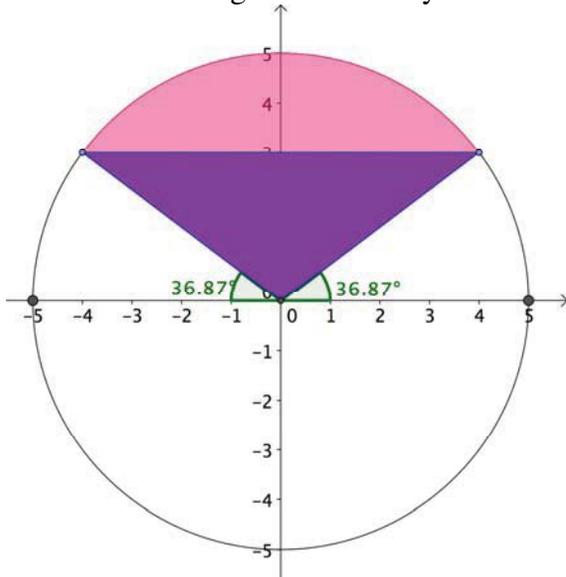
Area of a Sector:

We can find a formula for the area of a sector by setting up a ratio (using radian measures for angles, of course!):

$$\frac{A_{\text{sector}}}{A_{\text{circle}}} = \frac{\text{central angle}}{\text{full rotation}} \Rightarrow \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

So, after cross multiplying and simplifying, the area of a sector = _____

A pretty common problem involving sectors and triangles (specifically isosceles triangles) is to find the area inside a sector but outside the triangle formed by the radii and the chord connecting the end points of the circle. That’s a lot of words. Basically it amounts to finding the less darkly shaded region. You’ll need some trig to do it.



There is a way to generalize the solution of this problem. Let the central angle involved be θ and the radius be r . Find a general solution to the problem above.

See *TPC Sector & Triangle.ggb* for problems

The notes above roughly correlate to Section 3.4 in your textbook.

Name: _____

Date: _____

Period: _____

Trig- ***3.4 Arc Length and Area of a Sector***

Directions: Draw a picture for each, use the correct formula, and solve. For the arc length and area of a sector problems- make sure θ is in radians!! Otherwise the answers will be incorrect!!

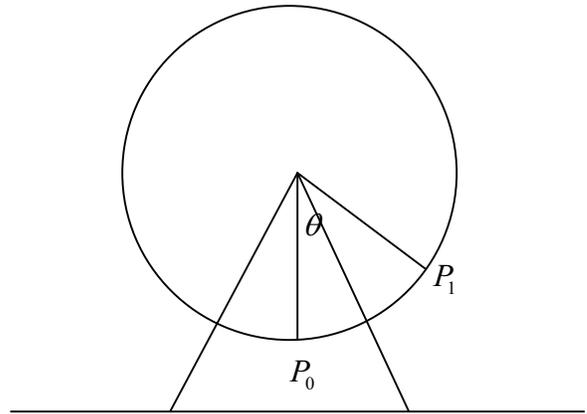
1. The minute hand of a clock is 3.8 cm long. How far does the tip of the minute hand travel in 25 minutes?

2. If the sector formed by a central angle of 30° has an area of $\frac{\pi}{6} \text{ cm}^2$, find the radius of the circle.

3. A lawn sprinkler is located at the corner of a yard. The sprinkler is set to rotate through 90° and project water out 100 feet. What is the area of the yard watered by the sprinkler?

4. The pendulum on a grandfather clock swings from side to side once every second. If the length of the pendulum is 3.5 feet and the angle through which it swings is 40° , how far does the tip of the pendulum travel in 1 second? 1 minute?

5. The Great Wheel is a Ferris wheel that was built in Vienna in 1897. The diameter of this wheel is 197 feet. Find the distance traveled by a rider in going from initial position P_0 to position P_1 , if:
- $\theta = 60^\circ$
 - $\theta = 210^\circ$
 - $\theta = 285^\circ$



6. An arc of length 9 feet is cut off by a central angle of $\frac{5\pi}{4}$ radians. Find the area of the sector formed.
7. An automobile windshield wiper 15 inches long rotates through an angle of 45° . If the rubber part of the blade covers only the last 12 inches of the wiper, find the area of the windshield cleaned by the windshield wiper.
8. If a 75-foot flagpole casts a shadow 43 feet long, what is the angle of elevation of the sun from the tip of the shadow?
9. A boat travels on a course of bearing $S 63^\circ 50' E$ for 114 miles. How many miles south and how many miles east has the boat traveled?

Name: _____

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Trig- ***3.4 Arc Length and Area of a Sector***

Directions: Draw a picture for each, use the correct formula, and solve. For the arc length and area of a sector problems- make sure θ is in radians!! Otherwise the answers will be incorrect!!

1. The minute hand of a clock is 3.8 cm long. How far does the tip of the minute hand travel in 25 minutes?

Answer: 9.9 cm

2. If the sector formed by a central angle of 30° has an area of $\frac{\pi}{6}$ cm², find the radius of the circle.

Answer: 1.4 cm

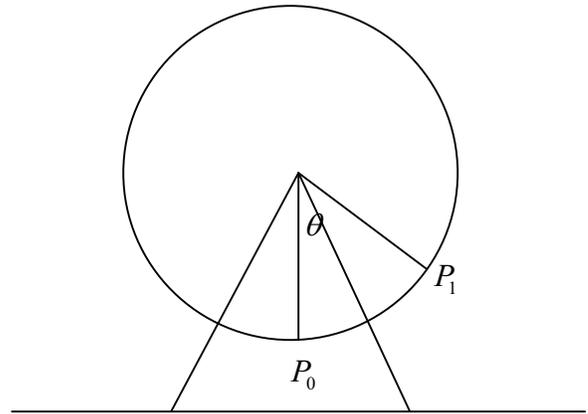
3. A lawn sprinkler is located at the corner of a yard. The sprinkler is set to rotate through 90° and project water out 100 feet. What is the area of the yard watered by the sprinkler? Answer: 7854 ft squared

4. The pendulum on a grandfather clock swings from side to side once every second. If the length of the pendulum is 3.5 feet and the angle through which it swings is 40° , how far does the tip of the pendulum travel in 1 second? 1 minute?

Answer: 2.4 feet (in 1 sec) and 144 ft in 60 sec or 1 min

5. The Great Wheel is a Ferris wheel that was built in Vienna in 1897. The diameter of this wheel is 197 feet. Find the distance traveled by a rider in going from initial position P_0 to position P_1 , if:
- $\theta = 60^\circ$
 - $\theta = 210^\circ$
 - $\theta = 285^\circ$

answers: a. 103 ft b. 361 ft. c. 490 ft.



6. An arc of length 9 feet is cut off by a central angle of $\frac{5\pi}{4}$ radians. Find the area of the sector formed.

Answer: 10.3 ft squared

7. An automobile windshield wiper 15 inches long rotates through an angle of 45° . If the rubber part of the blade covers only the last 12 inches of the wiper, find the area of the windshield cleaned by the windshield wiper.

Answer: 84.87 inches squared

8. If a 75-foot flagpole casts a shadow 43 feet long, what is the angle of elevation of the sun from the tip of the shadow? Answer: 60.2°

9. A boat travels on a course of bearing $S 63^\circ 50' E$ for 114 miles. How many miles south and how many miles east has the boat traveled? Answer: 50.3 miles south and 102.3 miles east