

From this last line we see that the keys to press are:

Scientific Calculator1.6003 $\boxed{1/x}$ $\boxed{\tan^{-1}}$ **Graphing Calculator** $\boxed{\tan^{-1}}$ $\boxed{(}$ 1.6003 $\boxed{x^{-1}}$ $\boxed{)}$ $\boxed{\text{ENTER}}$

To the nearest degree, the reference angle is $\hat{\theta} = 32^\circ$. Since we want θ to terminate in QII, we subtract 32° from 180° to get $\theta = 148^\circ$.

Again, we can check our result on a calculator by entering 148° , finding its tangent, and then finding the reciprocal of the result.

Scientific Calculator148 $\boxed{\tan}$ $\boxed{1/x}$ **Graphing Calculator**1 $\boxed{\div}$ $\boxed{\tan}$ $\boxed{(}$ 148 $\boxed{)}$ $\boxed{\text{ENTER}}$

The calculator gives a result of -1.6003 .

GETTING READY FOR CLASS

After reading through the preceding section, respond in your own words and in complete sentences.

- Define reference angle.
- State the reference angle theorem.
- What is the first step in finding the exact value of $\cos 495^\circ$?
- Explain how to find θ to the nearest tenth of a degree, if $\tan \theta = -0.8541$ and θ terminates in QIV with $0^\circ < \theta < 360^\circ$.

PROBLEM SET 3.1

Draw each of the following angles in standard position and then name the reference angle.

- | | | | |
|------------------|------------------|--------------------|--------------------|
| 1. 210° | 2. 150° | 3. 143.4° | 4. 253.8° |
| 5. 311.7° | 6. 93.2° | 7. $195^\circ 10'$ | 8. $171^\circ 40'$ |
| 9. -300° | 10. -330° | 11. -120° | 12. -150° |

Find the exact value of each of the following.

- | | | | |
|----------------------|----------------------|----------------------|----------------------|
| 13. $\cos 225^\circ$ | 14. $\cos 135^\circ$ | 15. $\sin 120^\circ$ | 16. $\sin 210^\circ$ |
| 17. $\tan 135^\circ$ | 18. $\tan 315^\circ$ | 19. $\cos 240^\circ$ | 20. $\cos 150^\circ$ |
| 21. $\csc 330^\circ$ | 22. $\sec 330^\circ$ | 23. $\sec 300^\circ$ | 24. $\csc 300^\circ$ |
| 25. $\sin 390^\circ$ | 26. $\cos 420^\circ$ | 27. $\cot 480^\circ$ | 28. $\cot 510^\circ$ |

Use a calculator to find the following.

- | | | | |
|--------------------------|--------------------------|--------------------------|--------------------------|
| 29. $\cos 347^\circ$ | 30. $\cos 238^\circ$ | 31. $\sec 101.8^\circ$ | 32. $\csc 166.7^\circ$ |
| 33. $\tan 143.4^\circ$ | 34. $\tan 253.8^\circ$ | 35. $\sec 311.7^\circ$ | 36. $\csc 93.2^\circ$ |
| 37. $\cot 390^\circ$ | 38. $\cot 420^\circ$ | 39. $\csc 575.4^\circ$ | 40. $\sec 590.9^\circ$ |
| 41. $\cos (-315^\circ)$ | 42. $\sin (-225^\circ)$ | 43. $\tan 195^\circ 10'$ | 44. $\tan 171^\circ 40'$ |
| 45. $\sec 314^\circ 40'$ | 46. $\csc 670^\circ 20'$ | 47. $\sin (-120^\circ)$ | 48. $\cos (-150^\circ)$ |

Use a calculator to find θ to the nearest tenth of a degree, if $0^\circ < \theta < 360^\circ$ and

- | | |
|---|---|
| 49. $\sin \theta = -0.3090$ with θ in QIII | 50. $\sin \theta = -0.3090$ with θ in QIV |
| 51. $\cos \theta = -0.7660$ with θ in QII | 52. $\cos \theta = -0.7660$ with θ in QIII |
| 53. $\tan \theta = 0.5890$ with θ in QIII | 54. $\tan \theta = 0.5890$ with θ in QI |
| 55. $\cos \theta = 0.2644$ with θ in QI | 56. $\cos \theta = 0.2644$ with θ in QIV |

First find
Reference
then add
180

57. $\sin \theta = 0.9652$ with θ in QII
 59. $\sec \theta = 1.4325$ with θ in QIV
 61. $\csc \theta = 2.4957$ with θ in QII
 63. $\cot \theta = -0.7366$ with θ in QII
 65. $\sec \theta = -1.7876$ with θ in QIII

Find θ , if $0^\circ < \theta < 360^\circ$ and

67. $\sin \theta = -\frac{\sqrt{3}}{2}$ and θ in QIII
 69. $\cos \theta = -\frac{1}{\sqrt{2}}$ and θ in QII
 71. $\sin \theta = -\frac{\sqrt{3}}{2}$ and θ in QIV

73. $\tan \theta = \sqrt{3}$ and θ in QIII

75. $\sec \theta = -2$ with θ in QII
 77. $\csc \theta = \sqrt{2}$ with θ in QII
 79. $\cot \theta = -1$ with θ in QIV

58. $\sin \theta = 0.9652$ with θ in QI
 60. $\csc \theta = 1.4325$ with θ in QII
 62. $\sec \theta = -3.4159$ with θ in QII
 64. $\cot \theta = -0.1234$ with θ in QIV
 66. $\csc \theta = -1.7876$ with θ in QIII

68. $\sin \theta = -\frac{1}{\sqrt{2}}$ and θ in QIII

70. $\cos \theta = -\frac{\sqrt{3}}{2}$ and θ in QIII

72. $\sin \theta = \frac{1}{\sqrt{2}}$ and θ in QII

74. $\tan \theta = \frac{1}{\sqrt{3}}$ and θ in QIII

76. $\csc \theta = 2$ with θ in QII
 78. $\sec \theta = \sqrt{2}$ with θ in QIV
 80. $\cot \theta = \sqrt{3}$ with θ in QIII

REVIEW PROBLEMS

The problems that follow review material we covered in Sections 1.1 and 2.1. Give the complement and supplement of each angle.

81. 70° 82. 120° 83. x 84. $90^\circ - y$

85. If the longest side in a 30° - 60° - 90° triangle is 10, find the length of the other two sides.
 86. If the two shorter sides of a 45° - 45° - 90° triangle are both $\frac{3}{4}$, find the length of the hypotenuse.

Simplify each expression by substituting values from the table of exact values and then simplifying the resulting expression.

87. $\sin 30^\circ \cos 60^\circ$ 88. $4 \sin 60^\circ - 2 \cos 30^\circ$
 89. $\sin^2 45^\circ + \cos^2 45^\circ$ 90. $(\sin 45^\circ + \cos 45^\circ)^2$

SECTION 3.2 RADIANs AND DEGREES

Although James Thomson is credited with the first printed use of the term *radian*, it is believed that the concept of radian measure was originally proposed by Roger Cotes (1682–1716), who was also the first to calculate 1 radian in degrees.

If you think back to the work you have done with functions of the form $y = f(x)$ in your algebra class, you will see that the variables x and y were always real numbers. The trigonometric functions we have worked with so far have had the form $y = f(\theta)$, where θ is measured in degrees. In order to apply the knowledge we have about functions from algebra to our trigonometric functions, we need to write our angles as real numbers, not degrees. The key to doing this is called *radian measure*. Radian measure is a relatively new concept in the history of mathematics. The term *radian* was first introduced by physicist James T. Thomson in examination questions in 1873. The introduction of radian measure will allow us to do a number of useful things. For instance, in Chapter 4 we will graph the function $y = \sin x$ on a rectangular coordinate system.