Exercise Set 1.3: Transformations of Graphs

Suppose that a student looks at a transformation of $y = f(x)$ and breaks it into the following steps. State the transformation that occurs in each step below.

1. $y = 2f(-x-3)+4$

    (a) From: $y = f(x)$ To: $y = f(x-3)$

    (b) From: $y = f(x-3)$ To: $y = f(-x-3)$

    (c) From: $y = f(-x-3)$ To: $y = 2f(-x-3)$

    (d) From: $y = 2f(-x-3)$
             To: $y = 2f(-x-3)+4$

**2.** $y = -\frac{1}{4}f(1-x)-5$

    First notice that $(1-x)$ is equivalent to $(-x+1)$

    (a) From: $y = f(x)$
           To: $y = f(x+1)$

    (b) From: $y = f(x+1)$
           To: $y = f(-x+1) = f(1-x)$

    (c) From: $y = f(1-x)$
           To: $y = \frac{1}{4}f(1-x)$

    (d) From: $y = \frac{1}{4}f(1-x)$
           To: $y = -\frac{1}{4}f(1-x)$

    (e) From: $y = -\frac{1}{4}f(1-x)$
   To: $y = -\frac{1}{4}f(1-x)-5$

Answer the following.

3. Jack and Jill are graphing the function $f(x) = 2 - x^2$. Starting with the graph of $y = x^2$, Jack first reflects the graph in the *x*-axis and then shifts upward two units. Jill, on the other hand, first shifts the graph $y = x^2$ upward two units and then reflects in the *x*-axis. Their graphs are shown below.
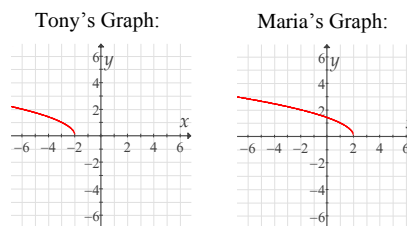
    Jack's Graph: Jill's Graph:
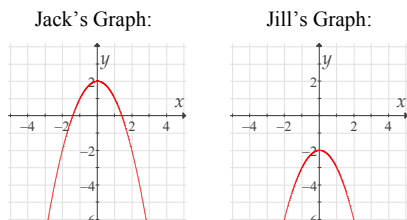
(a) Who is correct, Jack or Jill?

(b) Analyze the two methods and explain the algebraic difference between the two. Use this analysis to justify your answer in part (a).

4. Fred and Wilma are graphing the function $f(x) = (-x+4)^2$. Starting with the graph of $y = x^2$, Fred first reflects the graph in the *y*-axis and then shifts four units to the left. Wilma, on the other hand, first shifts the graph $y = x^2$ four units to the left and then reflects in the *y*-axis. Their graphs are shown below.

    Fred's Graph: Wilma's Graph:
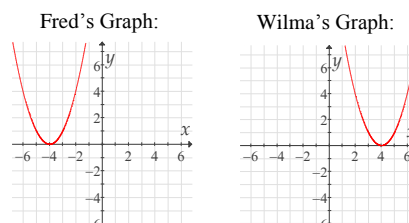
(a) Who is correct, Fred or Wilma?

(b) Analyze the two methods and explain the algebraic difference between the two. Use this analysis to justify your answer in part (a).

5. Tony and Maria are graphing the function $f(x) = \sqrt{-2-x}$. Starting with the graph of $y = \sqrt{x}$, Tony first shifts the graph two units to the right and then reflects in the *y*-axis. Maria, on the other hand, first reflects the graph $y = \sqrt{x}$ in the *y*-axis and then shifts two units to the right. Their graphs are shown below.

    Tony's Graph: Maria's Graph:

(a) Who is correct, Tony or Maria?

(b) Analyze the two methods and explain the algebraic difference between the two. Use this analysis to justify your answer in part (a).

6. Bart and Lisa are graphing the function $f(x) = 2|x| - 3$. Starting with the graph of $y = |x|$, Bart first shifts the graph downward three units and then stretches the graph vertically by a factor of 2. Lisa, on the other hand, first stretches the graph $y = |x|$ vertically by a factor of 2 and then shifts the graph downward three units.

Bart's Graph:	Lisa's Graph:
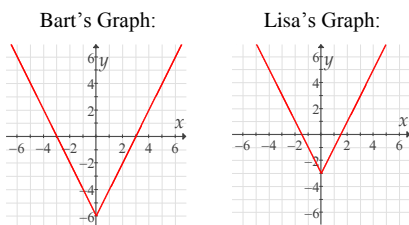
(a) Who is correct, Bart or Lisa?

(b) Analyze the two methods and explain the algebraic difference between the two. Use this analysis to justify your answer in part (a).

Matching. The left-hand column contains equations that represent transformations of $f(x) = x^2$. Match the equations on the left with the description on the right of how to obtain the graph of g from the graph of f.

7. $g(x) = (x-4)^2$

    A. Reflect in the x-axis.

8. $g(x) = x^2 - 4$

    B. Shift left 4 units, then reflect in the y-axis.

9. $g(x) = x^2 + 4$

    C. Reflect in the x-axis, then shift downward 4 units.

10. $g(x) = (x+4)^2$

    D. Shift right 4 units.

11. $g(x) = -x^2$

    E. Shift right 3 units, then reflect in the x-axis, then shift upward 4 units.

12. $g(x) = (-x)^2$

13. $g(x) = 4x^2$

    F. Shift upward 4 units.

    G. Reflect in the y-axis.

14. $g(x) = \frac{1}{4}x^2$

    H. Shift left 4 units, then shift upward 3 units.

15. $g(x) = -x^2 - 4$

16. $g(x) = (x+4)^2 + 3$

    I. Shift left 4 units.

17. $g(x) = -(x-3)^2 + 4$

    J. Shift downward 4 units.

18. $g(x) = (-x+4)^2$

    K. Stretch vertically by a factor of 4.

    L. Shrink vertically by a factor of $\frac{1}{4}$.

Write the equation that results when the following transformations are applied to the given standard function. Then state if any of the resulting functions in (a)-(e) are equivalent.

19. Standard function: $y = x^3$

(a) Shift right 7 units, then reflect in the x-axis, then stretch vertically by a factor of 5, then shift upward 1 unit.

(b) Reflect in the x-axis, then shift right 7 units, then stretch vertically by a factor of 5, then shift upward 1 unit.

(c) Stretch vertically by a factor of 5, then shift upward 1 unit, then shift right 7 units, then reflect in the x-axis.

(d) Shift right 7 units, then shift upward 1 unit, then reflect in the x-axis, then stretch vertically by a factor of 5.

(e) Reflect in the x-axis, then shift left 7 units, then stretch vertically by a factor of 5, then shift upward 1 unit.

(f) Which, if any, of the resulting functions in (a)-(e) are equivalent?

20. Standard function: $y = \sqrt{x}$

(a) Reflect in the y-axis, then shift left 2 units, then shift downward 4 units, then reflect in the x-axis

(b) Shift left 2 units, then reflect in the y-axis, then reflect in the x-axis, then shift downward 4 units.

(c) Reflect in the y-axis, then reflect in the x-axis, then shift downward 4 units, then shift right 2 units.

(d) Reflect in the x-axis, then shift left 2 units, then shift downward 4 units, then reflect in the y-axis.

(e) Shift downward 4 units, then shift left 2 units, then reflect in the y-axis, then reflect in the x-axis.

(f) Which, if any, of the resulting functions in (a)-(e) are equivalent?

Exercise Set 1.3: Transformations of Graphs

Describe how the graph of *g* is obtained from the graph of *f*. (Do not sketch the graph.)

21. $f(x) = \sqrt{x}$, $\qquad g(x) = \sqrt{-x} - 2$

22. $f(x) = x^3$, $\qquad g(x) = -2(x+5)^3$

23. $f(x) = |x|$, $\qquad g(x) = -5|x-2| + 1$

24. $f(x) = x^2$, $\qquad g(x) = \frac{1}{6}(x+3)^2 - 7$

25. $f(x) = \frac{1}{x}$, $\qquad g(x) = \frac{3}{x+8} + 2$

26. $f(x) = \sqrt[3]{x}$, $\qquad g(x) = \sqrt[3]{-x} + 4$

Describe how the graphs of each of the following functions can be obtained from the graph of *y = f(x)*.

27. $y = f(x) + 1$

28. $y = f(x - 7)$

29. $y = f(-x) + 3$

30. $y = -f(x+3) - 8$

31. $y = -\frac{1}{4}f(x-2) - 5$

32. $y = -5f(-x) + 1$

33. $y = f(7 - x) + 2$

34. $y = f(-x - 5) - 7$

Sketch the graph of each of the following functions. Do not plot points, but instead apply transformations to the graph of a standard function.

35. $f(x) = x - 3$

36. $f(x) = 5 - x$

37. $f(x) = -3x + 1$

38. $f(x) = 2x - 7$

39. $f(x) = x^2 + 3$

40. $f(x) = (x - 5)^2$

41. $f(x) = 6 - x^2$

42. $f(x) = 2 - (x - 1)^2$

43. $f(x) = -3(x - 4)^2 - 2$

44. $f(x) = (x + 5)^2 + 3$

45. $f(x) = 6 - \sqrt{x+2}$

46. $f(x) = \frac{1}{2}\sqrt{-x} + 1$

47. $f(x) = \sqrt{-x+4} + 2$

48. $f(x) = \sqrt{5-x} - 1$

49. $f(x) = 2|x+5| - 3$

50. $f(x) = -|x-2| + 4$

51. $f(x) = -(x-4)^3 + 1$

52. $f(x) = -x^3 - 5$

53. $f(x) = \frac{1}{x-3} + 6$

54. $f(x) = -\frac{2}{x+4}$

55. $f(x) = -\frac{4}{x} + 3$
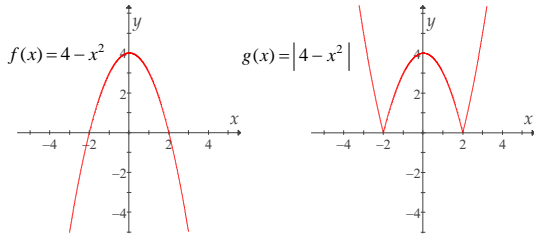
56. $f(x) = \sqrt[3]{x-6}$

57. $f(x) = \sqrt[3]{-x} + 2$
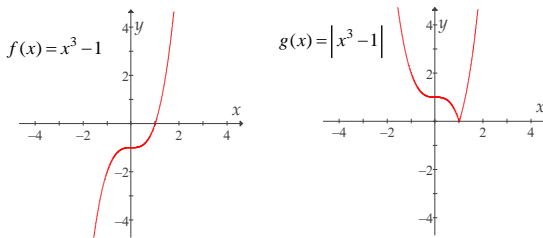
58. $f(x) = -\sqrt[3]{x+1} - 5$

Exercise Set 1.3: Transformations of Graphs

Answer the following.

59. The graphs of $f(x) = 4 - x^2$ and $g(x) = \left|4 - x^2\right|$ are shown below. Describe how the graph of g was obtained from the graph of f.
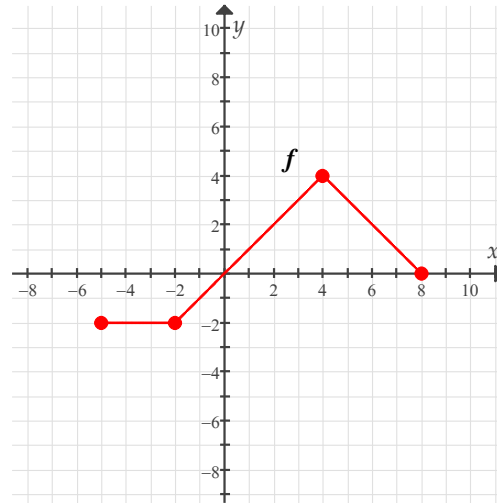

60. The graphs of $f(x) = x^3 - 1$ and $g(x) = \left|x^3 - 1\right|$ are shown below. Describe how the graph of g was obtained from the graph of f.

Sketch the graphs of the following functions:

61. (a) $f(x) = x^2 - 9$ (b) $g(x) = \left|x^2 - 9\right|$

62. (a) $f(x) = \dfrac{1}{x}$ (b) $g(x) = \left|\dfrac{1}{x}\right|$

Continued in the next column…

The graph of $y = f(x)$ is given below. Sketch the graph of each of the following functions.

63. $y = f(x + 2)$

64. $y = f(x) - 3$

65. $y = f(x - 2) - 1$

66. $y = f(x + 1) + 5$

67. $y = f(-x)$

68. $y = -f(x)$

69. $y = 2f(x)$

70. $y = \frac{1}{2}f(x)$

71. $y = -2f(x + 1)$

72. $y = f(-x) - 4$

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