PROBLEM SET 1.2

Graph each of the following ordered pairs on a rectangular coordinate system:

1. $(2, 4)$ **2.** $(2, -4)$ **3.** $(-2, 4)$ **4.** $(-2, -4)$
5. $(-3, -4)$ **6.** $(-4, -2)$ **7.** $(5, 0)$ **8.** $(-3, 0)$
9. $(0, -3)$ **10.** $(0, 2)$ **11.** $\left(-5, \frac{1}{2}\right)$ **12.** $\left(5, \frac{1}{2}\right)$

Graph each of the following lines:

13. $3x + 2y = 6$ **14.** $y = 2x - 1$ **15.** $y = \frac{1}{2}x$ **16.** $x = 3$

Graph each of the following parabolas:

17. $y = x^2 - 4$ **18.** $y = (x - 2)^2$
19. $y = (x + 2)^2 + 4$ **20.** $y = \frac{1}{4}(x + 2)^2 + 4$

21. Use your graphing calculator to graph $y = ax^2$ for $a = \frac{1}{10}, \frac{1}{5}$, 1, 5, and 10. Copy all five graphs onto a single coordinate system and label each one. What happens to the shape of the parabola as the value of a gets close to zero? What happens to the shape of the parabola when the value of a gets large?

22. Use your graphing calculator to graph $y = ax^2$ for $a = \frac{1}{5}$, 1, and 5, then again for $a = -\frac{1}{5}, -1$, and -5. Copy all six graphs onto a single coordinate system and label each one. Explain how a negative value of a affects the parabola.

23. Use your graphing calculator to graph $y = (x - h)^2$ for $h = -3$, 0, and 3. Copy all three graphs onto a single coordinate system and label each one. What happens to the position of the parabola when $h < 0$? What if $h > 0$?

24. Use your graphing calculator to graph $y = x^2 + k$ for $k = -3$, 0, and 3. Copy all three graphs onto a single coordinate system and label each one. What happens to the position of the parabola when $k < 0$? What if $k > 0$?

Graph each of the following circles:

25. $x^2 + y^2 = 25$ **26.** $x^2 + y^2 = 36$
27. $x^2 + y^2 = 5$ **28.** $x^2 + y^2 = 6$

Graph the circle $x^2 + y^2 = 1$ with your graphing calculator. Use the feature on your calculator that allows you to evaluate a function from the graph to find the coordinates of all points on the circle that have the given x-coordinate. Write your answers as ordered pairs and round to four places past the decimal point when necessary.

29. $x = \frac{1}{2}$ **30.** $x = -\frac{1}{2}$ **31.** $x = \frac{\sqrt{2}}{2}$
32. $x = -\frac{\sqrt{2}}{2}$ **33.** $x = -\frac{\sqrt{3}}{2}$ **34.** $x = \frac{\sqrt{3}}{2}$

35. Use the graph of Problem 25 to name the points at which the line $x + y = 5$ will intersect the circle $x^2 + y^2 = 25$.

36. Use the graph of Problem 26 to name the points at which the line $x - y = 6$ will intersect the circle $x^2 + y^2 = 36$.

37. At what points will the line $y = x$ intersect the unit circle $x^2 + y^2 = 1$?

38. At what points will the line $y = -x$ intersect the unit circle $x^2 + y^2 = 1$?

Find the distance between the following points:

39. $(3, 7), (6, 3)$ **40.** $(4, 7), (8, 1)$ **41.** $(0, 12), (5, 0)$
42. $(-3, 0), (0, 4)$ **43.** $(-1, -2), (-10, 5)$ **44.** $(-3, 8), (-1, 6)$

45. Find the distance from the origin out to the point $(3, -4)$.
46. Find the distance from the origin out to the point $(12, -5)$.
47. Find x so the distance between $(x, 2)$ and $(1, 5)$ is $\sqrt{13}$.
48. Find y so the distance between $(7, y)$ and $(3, 3)$ is 5.
49. **Pythagorean Theorem** An airplane is approaching Los Angeles International Airport at an altitude of 2,640 feet. If the horizontal distance from the plane to the runway is 1.2 miles, use the Pythagorean Theorem to find the diagonal distance from the plane to the runway (Figure 24). (5,280 feet equals 1 mile.)

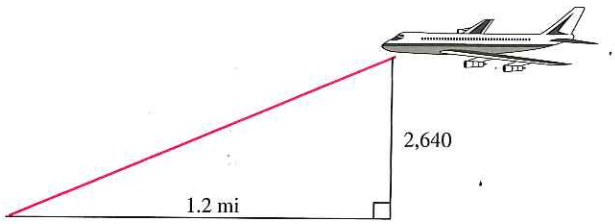

Figure 24

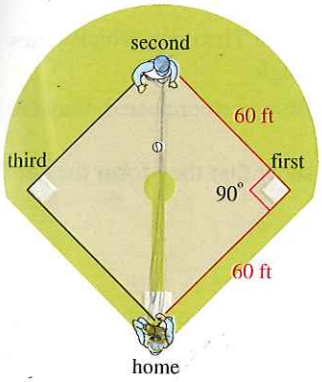

Figure 25

50. **Softball Diamond** In softball, the distance from home plate to first base is 60 feet, as is the distance from first base to second base. If the lines joining home plate to first base and first base to second base form a right angle, how far does a catcher standing on home plate have to throw the ball so that it reaches the shortstop standing on second base (Figure 25)?
51. **Softball and Rectangular Coordinates** If a coordinate system is superimposed on the softball diamond in Problem 50 with the x-axis along the line from home plate to first base and the y-axis on the line from home plate to third base, what would be the coordinates of home plate, first base, second base, and third base?
52. **Softball and Rectangular Coordinates** If a coordinate system is superimposed on the softball diamond in Problem 50 with the origin on home plate and the positive x-axis along the line joining home plate to second base, what would be the coordinates of first base and third base?
53. In what two quadrants do all the points have negative x-coordinates?
54. In what two quadrants do all the points have negative y-coordinates?
55. For points (x, y) in quadrant I, the ratio x/y is always positive because x and y are always positive. In what other quadrant is the ratio x/y always positive?
56. For points (x, y) in quadrant II, the ratio x/y is always negative because x is negative and y is positive in quadrant II. In what other quadrant is the ratio x/y always negative?
57. **Human Cannonball** A human cannonball is shot from a cannon at the county fair. He reaches a height of 60 feet before landing in a net 160 feet from the cannon. Sketch the graph of his path, and then find the equation of the graph. Verify that your equation is correct using your graphing calculator.
58. **Human Cannonball** Referring to Problem 57, find the height reached by the human cannonball after he has traveled 30 feet horizontally, and after he has traveled 150 feet horizontally. Verify that your answers are correct using your graphing calculator.

Use the diagram in Figure 23 to help find the complement of each of the following angles.

59. 45° **60.** 30° **61.** 60° **62.** 0°

Use the diagram in Figure 23 to help find the supplement of each of the following angles.

63. 120° **64.** 150° **65.** 90° **66.** 45°

Use the diagram in Figure 23 to help you name an angle between 0° and 360° that is coterminal with each of the following angles.

67. $-60°$ **68.** $-135°$ **69.** $-210°$ **70.** $-300°$

Draw each of the following angles in standard position and then do the following:

a. Name a point on the terminal side of the angle.

b. Find the distance from the origin to that point.

c. Name another angle that is coterminal with the angle you have drawn.

71. 135° **72.** 45° **73.** 225° **74.** 315°

75. 90° **76.** 360° **77.** $-45°$ **78.** $-90°$

79. Draw an angle of 30° in standard position. Then find a if the point $(a, 1)$ is on the terminal side of 30°.

80. Draw 60° in standard position. Then find b if the point $(2, b)$ is on the terminal side of 60°.

81. Draw an angle in standard position whose terminal side contains the point $(3, -2)$. Find the distance from the origin to this point.

82. Draw an angle in standard position whose terminal side contains the point $(2, -3)$. Find the distance from the origin to this point.

For problems 83 and 84, use the converse of the Pythagorean Theorem, which states that if $c^2 = a^2 + b^2$, then the triangle must be a right triangle.

83. Plot the points (0, 0), (5, 0), and (5, 12) and show that, when connected, they are the vertices of a right triangle.

84. Plot the points (0, 2), $(-3, 2)$, and $(-3, -2)$ and show that they form the vertices of a right triangle.

EXTENDING THE CONCEPTS

85. Descartes and Pascal In the introduction to this section we mentioned two French philosophers, Descartes and Pascal. Many people see the philosophies of the two men as being opposites. Why is this?

86. Pascal's Triangle Pascal has a triangular array of numbers named after him, Pascal's triangle. What part does Pascal's triangle play in the expansion of $(a + b)^n$, where n is a positive integer?

SECTION 1.3 DEFINITION I: TRIGONOMETRIC FUNCTIONS

In this section we begin our work with trigonometry. The formal study of trigonometry dates back to the Greeks, when it was used mainly in the design of clocks and calendars and in navigation. The trigonometry of that period was spherical in nature, as it was based on measurement of arcs and chords associated with spheres. Unlike the trigonometry of the Greeks, our introduction to trigonometry takes place on a rectangular coordinate system. It concerns itself with angles, line segments, and points in the plane.

The definition of the trigonometric functions that begins this section is one of three definitions we will use. For us, it is the most important definition in the book. What should you do with it? Memorize it. Remember, in mathematics, definitions are simply accepted. That is, unlike theorems, there is no proof associated with a definition; we simply accept them exactly as they are written, memorize them, and then use them. When you are finished with this section, be sure you have memorized this first definition. It is the most valuable thing you can do for yourself at this point in your study of trigonometry.