Pre-Calculus **Name** MR. SCEVOLA
Review -- Ch 15.5 **Period** _____ **Date** _____

1. The polynomial $a+b$ is called a binomial because it has two terms.

2. Consecutive powers of $a+b$ [e.g., $(a+b)^1, (a+b)^2, (a+b)^3$, etc.] are referred to collectively as the binomial Expansion.

3. The binomial Theorem tells us that for any positive integer n
$(a+b)^n = {}_nC_0 a^n b^0 + {}_nC_1 a^{n-1} b^1 + {}_nC_2 a^{n-2} b^2 + \cdots + {}_nC_n a^0 b^n$

4. ${}_nC_r =$ $\frac{n!}{r!(n-r)!}$ in factorial notation.

5. For any positive integer n, ${}_nC_0 =$ 1 ; ${}_nC_1 =$ n ; ${}_nC_n =$ 1

6. $a^0 =$ 1 ; $b^1 =$ b ; $a^m a^n =$ a^{m+n} ; $\frac{a^m}{a^n} =$ a^{m-n} ; $(a^m)^n =$ a^{mn} ; $(-1)^n =$ 1 if n even, −1 if n odd

7. Complete Pascal's Triangle through n = 10:

n	Coefficients of $(a+b)^n$
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1
7	1 7 21 35 35 21 7 1
8	1 8 28 56 70 56 28 8 1
9	1 9 36 84 126 126 84 36 9 1
10	1 10 45 120 210 252 210 120 45 10 1

8. For the expansion of $(a+b)^n$ where n≥5, provide the following:

   a. The coefficient of the 5th term is [redacted] nC5 [redacted]

   b. The exponent of a in the 5th term is 5

   c. The exponent of b in the 5th term is n−5

   d. For any term, the sum of the exponents of a and b is always n

   e. The entire 5th term is $(nC5)a^5 b^{(n-5)}$

9. Expand each of the following:

a. $(x^2-y^2)^3 = {}_3C_0(x^2)^3(-y)^0 + {}_3C_1(x^2)^2(-y)^1 + {}_3C_2(x^2)(-y)^2 + {}_3C_3(x^2)^0(-y)^3$

$1 \cdot x^6 + 3x^4(-y) + 3x^2y^2 + 1(-y^3)$

$x^6 - 3x^4y + 3x^2y^2 - y^3$

b. $(2x^2-1)^4 = {}_4C_0(2x^2)^4(-1)^0 + {}_4C_1(2x^2)^3(-1)^1 + {}_4C_2(2x^2)^2(-1)^2 + {}_4C_3(2x^2)^1(-1)^3 + {}_4C_4(+2x)^0(-1)^4$

$1(16x^8) + 4(8x^6)(-1) + 6(4x^4)(1) + 4(2x^2)(-1) + 1(1)$

$16x^8 - 32x^6 + 24x^4 + 8x^2 + 1$

c. $(x^2+2)^3 = {}_3C_0(x^2)^3(2)^0 + {}_3C_1(x^2)^2(2)^1 + {}_3C_2(x^2)^1(2)^2 + {}_3C_3(x^2)^0(2)^3$

$1 \cdot x^6 + 3(x^4)2 + 3(x^2)(4) + 8 = x^6 + 6x^4 + 12x^2 + 8$

10. In the expansion of $(a+b)^{15}$, find

a. The 9th term ${}_{15}C_9 a^9 b^6 = 5005a^9b^6$

b. The 10th term ${}_{15}C_{10} a^{10} b^5 = 3003a^{10}b^5$

11. In the expansion of $(a-b)^{15}$, find:

a. The 9th term ${}_{15}C_9(a)^9(-b)^6 = 5005a^9b^6$

b. The 10th term ${}_{15}C_{10}(a)^{10}(-b)^5 = -3003a^{10}b^5$

12. Find the 8th term of $(p+q)^{50}$ ${}_{50}C_8(p)^8(q)^{42} = 536878650p^8q^{42}$

13. Find the coefficient of x^2 in the expansion of $\left(x-\frac{1}{x}\right)^{10}$ ${}_{10}C_2(x)^2\left(-\frac{1}{x}\right)^8$

$45x^2\left(\frac{1}{x^8}\right) = \frac{45}{x^6}$

14. Find the coefficient of x^{10} in the expansion of $(x^2-1)^9$ ${}_{10}C_5(x^2)^5(-1)^4$

$252x^{10}$