d it easier to owing princi-



least one digit 5.

least one digit 5, 5's.

a license plate

many different

se-and-sweater

r, but not both.

s can prizes be

nt of the letters

me row" of a possible?

words rather

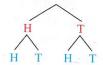
## WRITTEN EXERCISES

- - 1. Evaluate: a. 5!
- **b.** 6!
- c. 7! d. 0!
- **2.** Evaluate: **a.**  $\frac{10!}{9!}$  **b.**  $\frac{20!}{18!}$  **c.**  $\frac{n!}{(n-1)!}$  **d.**  $\frac{(n+1)!}{(n-1)!}$

- 3. In how many different orders can you arrange 5 books on a shelf?
- 4. In how many different orders can 9 people stand in a line?
- 5. In how many different ways can you answer 10 true-false questions?
- 6. In how many different ways can you answer 10 multiple-choice questions if each question has 5 choices?
- 7. Many radio stations have 4-letter call signs beginning with K. How many such call signs are possible if letters (a) can be repeated? (b) cannot be repeated?
- 8. How many 3-digit numbers can be formed using the digits 4, 5, 6, 7,8 if the digits (a) can be repeated? (b) cannot be repeated?
- 9. In how many ways can 4 people be seated in a row of 12 chairs?



- 10. In how many ways can 4 different prizes be given to any 4 of 10 people if no person receives more than 1 prize?
- 11. The tree diagram shows the four possible outcomes when a coin is tossed twice. If H and T represent "heads" and "tails," respectively, then the four outcomes are HH, HT, TH, and TT.



- a. Make a tree diagram showing the outcomes if a coin is tossed 3 times.
- b. The solid lines of the tree diagram are called branches and the elements of the bottom row (H, T, H, and T) are called leaves. How many branches and leaves are in your diagram for part (a)?
- c. How many branches and leaves would there be in a tree diagram showing the toss of a coin 10 times?
- 12. Four cards numbered 1 through 4 are shuffled and 3 different cards are chosen one at a time. Make a tree diagram showing the various possible outcomes.
- B 13. Sports A high school coach must decide on the batting order for a baseball team of 9 players.
  - a. The coach has how many different batting orders from which to choose?
  - b. How many different batting orders are possible if the pitcher bats last?
  - c. How many different batting orders are possible if the pitcher bats last and the team's best hitter bats third?

- 4-person 400 m relay team and a 4-person 800 m relay team from a squad of 7 sprinters, any of whom can run on either team. If the fastest sprinter runs last in both races, in how many ways can the coach form the two teams if each of the 6 remaining sprinters runs only once and each different order is counted as a different team?
- 15. How many numbers consisting of 1, 2, or 3 digits (without repetitions) can be formed using the digits 1, 2, 3, 4, 5, 6?



- 16. If you have 5 signal flags and can send messages by hoisting one or more flags on a flagpole, how many messages can you send?
- 17. In some states license plates consist of 3 letters followed by 2 or 3 digits (for example, RRK-54 or ABC-055). How many such possibilities are there for those plates with 2 digits? for those with 3 digits? In all, how many license plates are possible?
- **18.** How many possibilities are there for a license plate with 2 letters and 3 or 4 nonzero digits?
- 19. a. How many 3-digit numbers contain no 7's?
  - b. How many 3-digit numbers contain at least one 7?
- 20. a. How many 4-digit numbers contain no 8's or 9's?b. How many 4-digit numbers contain at least one 8 or 9?
- 21. How many numbers from 5000 to 6999 contain at least one 3?
- 22. Many license plates in the U.S./consist of 3 letters followed by a 3-digit number from 100 to 999. How many of these contain at least one of the vowels A, E, I, O, and U?
- 23. Telephone numbers in the U.S. and Canada have 10 digits as follows:

3-digit area code number: first digit is not 0 or 1;

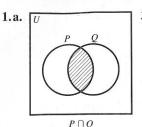
second digit must be 0 or 1

3-digit exchange number: first and second digits are *not* 0 or 1 4-digit line number: *not* all zeros

- a. How many possible area codes are there?
- **b.** The area code for Chicago is 312. Within this area code how many exchange numbers are possible?
- **c.** One of the exchange numbers for Chicago is 472. Within this exchange, how many line numbers are possible?
- d. How many 7-digit phone numbers are possible in the 312 area code?
- e. How many 10-digit phone numbers are possible in the U.S. and Canada?

**6.** The set of freshmen who are not music majors; 285 **7.** The set of students who are neither freshmen nor music majors; 650 **8.** The set of students who are either freshmen or are not music majors; 950 **9.a.**  $B \cap \overline{A}$  **b.**  $A \cap B$  **c.**  $\overline{A \cup B}$  or  $\overline{A} \cap \overline{B}$  **10.a.**  $A \in B$  **c.**  $\overline{A} \cup \overline{B}$  **c.**  $\overline{A} \cup \overline{B}$ 

## Written Exercises, pages 568-571



3.a. \( \begin{align\*} \begin{align\*

**5.a.** Teachers of either math or physics **b.** Teachers of math but not of physics **7.a.** Teachers of biology or of both physics and chemistry **b.** Teachers of both biology and chemistry **9.a.** Teachers of neither math nor physics nor chemistry **b.** Same as part a. **11.** 28 **13.** 12 **19.a.** 11 **b.** 12 **21.**  $\emptyset$  **25.b.**  $\overline{A \cap B} = \overline{A} \cup \overline{B}$  **c.**  $\overline{A} \cup (\overline{B} \cap \overline{C})$  **27.**  $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(B \cap C) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$ 

Class Exercises, page 574 1.a. 2 b. 6 c. 24 2. 3,628,800 3. 608,400 4.a. 60 b. 180 5.a. 8 b. 6 6. 720 7.a. 362,879 b. 1,133,836,704,000

Written Exercises, pages 575–577 1.a. 120 b. 720 c. 5040 d. 1 3. 120 5. 1024 7.a. 17,576 b. 13,800 9. 11,880 11.b. 14 branches, 8 leaves c. 2046 branches, 1024 leaves 13.a. 362,880 b. 40,320 c. 5040 15. 156 17. 2 digits: 1,757,600; 3 digits: 17,576,000; total: 19,333,600 19.a. 648 b. 252 21. 542 23.a. 160 b. 640 c. 9999 d. 6,399,360 e. 1,023,897,600 25.a. 362,880 b. 30,240 27.  $26^2 = 676 < 677$  31.  $\log_{10} 9! \approx 5.56$ ;  $\log_{10} 10! \approx 6.56$  33.a.  $\frac{10!}{6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6!} = 10 \cdot 9 \cdot 8 \cdot 7$ 

Class Exercises, page 580 1.a. 20 b. 10
2.a. 120 b. 20 3.a. 720 b. 120 4.a. 24 b. 1
5. 720 6. 120 7. The order of the three nos. is important.

8.a. ABC ACB BAC BCA CAB CBA
ABD ADB BAD BDA DAB DBA
ACD ADC CAD CDA DAC DCA
BCD BDC CBD CDB DBC DCB
b. A, B, and C: 6; A, C, and D: 6 c. ABC, ABD,
ACD, BCD

Written Exercises, pages 580-582 1.a. 380 b. 190 3.a. 210 b. 5040 5.a. 1,313,400 b. 7,880,400 7.a. 336 b. 56 9.a. 665,280 b. 924 11.a. 35 b. 35 13.a.  $_{100}C_2 = _{100}C_{98} = \frac{100!}{2! \, 98!}$  15.a. 1 b. 1 17. 22,764 19. 48 21.a. 792 b. 658,008 c. 1,940,952 23.a. 15 b. There will be fewer line segs. if three or more pts. are collinear. 25. 10 29.  $5.36 \times 10^{28}$ 

Class Exercises, page 585 1.a. 24 b. 12 2.a. 120 b. 60 3.a. 720 b. 180 4.a. 39,916,800 b. 1,663,200 7. 84 8. BCDAE

**Written Exercises, pages 585–587** 1. 10,080 **3.** 34,650 **5.** 840 **7.** 9.71  $\times$  10<sup>14</sup> years **9.a.** 3003 **b.** 21 **c.** 35 **d.** 735 11. 24 13.  $1.37 \times 10^{11}$  15.b.  $8.25 \times 10^{12}$  17. 6188

Mixed Combinatorics Exercises, pages 587–589
1. 161,700 3.a. 6561 b. 5832 c. 2000 5.a. 870
b. 435 7. 60 ways from A to D; 3600 round trips
9. 83 11.a. 1 min b. 13 h 13.a. 70 b. 35 c. 35
15. 15 17. 64 19.a. 86,400 b. 28,800
21. 37,512

Class Exercises, page 592
2. 1 7 21 35 35 21 7 1 3.a. 1 8 28 56
b.  $x^8$ ,  $8x^7y$ ,  $28x^6y^2$ ,  $56x^5y^3$ c.  $x^8$ ,  $-8x^7y$ ,  $28x^6y^2$ ,  $-56x^5y^3$ 4.  $a^3 - 3a^2b + 3ab^2 - b^3$ 5.  $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ 6.  $a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$ 7.a.  $15x^4y^2$ b.  $84x^6y^3$ 

Written Exercises, pages 592-594

1.a.  $a^3 + 3a^2b + 3ab^2 + b^3$ b. 8000 + 1200 + 60 + 1 = 9261c. 8000 - 1200 + 60 - 1 = 68593.a.  $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$ b.  $a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$ c.  $32a^5 + 80a^4 + 80a^3 + 40a^2 + 10a + 1$ 5.a.  $x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$  b.  $x^7 - 7x^6y + 21x^5y^2 - 35x^4y^3 + 35x^3y^4 - 21x^2y^5 + 7xy^6 - y^7$