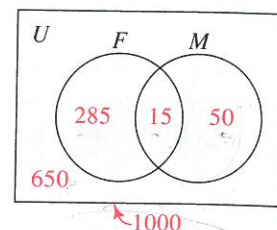


In Example 2 we found that the set $\overline{J \cup B \cup T}$ has no elements in it. Such a set is called the **empty set** and is denoted by the Greek letter ϕ (phi). Thus, $\overline{J \cup B \cup T} = \phi$.

CLASS EXERCISES

A certain small college has 1000 students. Let F = the set of college freshmen, and let M = the set of music majors. These sets are shown in the Venn diagram at the right. Describe each of the following sets in words and tell how many members it has.



1. $F \cap M$
2. $F \cup M$
3. \overline{F}
4. \overline{M}
5. $\overline{F} \cap M$
6. $F \cap \overline{M}$
7. $\overline{F \cup M}$
8. $F \cup \overline{M}$
9. **Reading** Refer to the first Venn diagram on page 565. People who have "type A" blood belong to the set $A \cap \overline{B}$. To what set do people with each of the following blood types belong?
 - a. B
 - b. AB
 - c. O
10. If A is any subset of a universal set U , complete the following.
 - a. $A \cup \phi = ?$
 - b. $A \cap \phi = ?$
 - c. $A \cup \overline{A} = ?$
 - d. $A \cap \overline{A} = ?$
 - e. $A \cup U = ?$
 - f. $A \cap U = ?$

WRITTEN EXERCISES

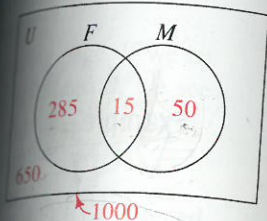
For Exercises 1–4, draw a Venn diagram and shade the region representing the set given in part (a). Then draw a separate Venn diagram and shade the region representing the set given in part (b).

- A**
1. a. $P \cap Q$
 2. a. $\overline{P} \cap Q$
 3. a. $\overline{P \cup Q}$
 4. a. $\overline{P \cap Q}$
 - b. $P \cup Q$
 - b. $P \cap \overline{Q}$
 - b. $\overline{P} \cap \overline{Q}$
 - b. $\overline{P} \cup \overline{Q}$

Let U = the universal set of all teachers in your school. Let the subsets of mathematics teachers, biology teachers, physics teachers, and chemistry teachers be represented by M , B , P , and C , respectively. Describe in words each of the following sets, and name a teacher belonging to each set if such a teacher exists in your school.

5. a. $M \cup P$
- b. $M \cap \overline{P}$
6. a. $P \cap C$
- b. $\overline{P \cup C}$
7. a. $B \cup (P \cap C)$
- b. $(B \cup P) \cap (B \cap C)$
8. a. $(B \cup C) \cap M$
- b. $(B \cap M) \cup (C \cap M)$

elements in it. Such a letter ϕ (phi). Thus,



9. a. \overline{MUPUC}
10. a. $\overline{M} \cap \overline{B} \cap \overline{P} \cap \overline{C}$

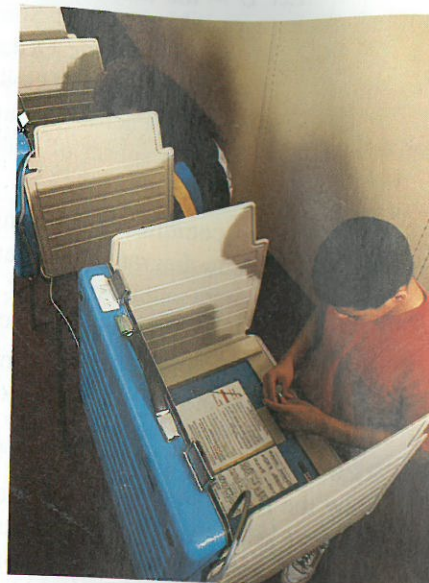
- b. $\overline{M} \cap \overline{P} \cap \overline{C}$
b. $\overline{MUBUPUC}$

In Exercises 11–14, draw a Venn diagram to illustrate each situation described. Then use the diagram to answer the question asked.

11. In an election-day survey of 100 voters leaving the polls, 52 said they voted for Proposition 1, and 38 said they voted for Proposition 2. If 18 said they voted for both, how many voted for neither?
12. Although the weather was perfect for the beach party, 17 of the 30 people attending got a sunburn and 25 people were bitten by mosquitoes. If 12 people were both bitten and sunburned, how many had neither affliction?
13. In a survey of 48 high school students, 20 liked classical music and 16 liked bluegrass music. Twenty students said they didn't like either. How many liked classical but not bluegrass? $16 + 20 - x = 28$
14. Of the 52 teachers at Roosevelt High School, 27 said they like to go sailing, 25 said they like to go fishing, and 12 said they don't enjoy either recreation. How many enjoy fishing but not sailing? $27 + 25 - x = 40$
- B** 15. **Astronomy** Consider the sets defined below. (You may need to consult an encyclopedia to determine the elements of each set.)

Let U = the universal set of planets in our solar system, 8 planets
 S = the subset of planets smaller than Earth,
and F = the subset of planets farther from the Sun than Earth.

- a. Draw a Venn diagram with overlapping circles representing S and F inside a rectangular region U . Inside each of the four regions of your diagram, list the planets described by that region.
- b. The smallest set to which the planet Venus belongs is $S \cap \overline{F}$. What is the smallest set to which the planet Uranus belongs?
16. **Geography** Consider the sets defined below.
- Let U = the universal set of states in the United States,
 P = the subset of states bordering the Pacific Ocean,
and M = the subset of states bordering Mexico.
- a. Name all the states in the set $\overline{P} \cap M$.
- b. What is the only state in the set $P \cap M$?
- c. How many states are in the set $\overline{P} \cup \overline{M}$?



$$21. \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}; R_{180}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 1 & 4 \end{bmatrix}$$

na, 30 Papa,
y; 1.5S;

$$\begin{bmatrix} 5 & 0 & -6 \\ -6 & 3 & 8 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix};$$

duct matrix XY
row of X and
e different for
4 2.8 3.4]

c Overlook,
to other sites.

$$7.a. \begin{matrix} c & s \\ c[0.92 & 0.08] \\ s[0.03 & 0.97] \end{matrix}$$

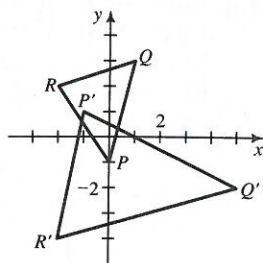
$$b. [0.6 \ 0.4]$$

c. City: 56.4%;
suburbs: 43.6%

8.a,b. See graph at right.

$$c. T = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix},$$

$|T| = -3$; $\frac{\text{area } \triangle P'Q'R'}{\text{area } \triangle PQR} = 3$ and the \triangle have
opp. orientations.



$$21.a. \begin{bmatrix} -4 & 7 \\ 15 & 8 \\ -9 & -4 \end{bmatrix}$$

$$b. AB = \begin{bmatrix} -30 & -7 \\ -6 & -3 \end{bmatrix},$$

$$BA = \begin{bmatrix} -5 & 8 & -4 \\ 3 & -18 & 6 \\ 7 & 46 & -10 \end{bmatrix}; \text{ no } 22. \begin{bmatrix} \frac{2}{3} & -\frac{5}{3} \\ 0 & 1 \end{bmatrix}$$

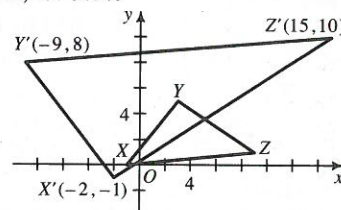
$$23. \begin{bmatrix} -2 \\ 2 \end{bmatrix} \quad 24.a. \begin{matrix} A & B & C & D & E \\ A \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \end{bmatrix} \\ B \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \end{bmatrix} \\ C \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \end{bmatrix} \\ D \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \end{bmatrix} \\ E \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$\begin{matrix} A & B & C & D & E \\ A \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \end{bmatrix} \\ B \begin{bmatrix} 1 & 1 & 2 & 2 & 2 \end{bmatrix} \\ b. C \begin{bmatrix} 0 & 1 & 2 & 2 & 2 \end{bmatrix} \\ D \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \\ E \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

$$\begin{matrix} A & B & C \\ A \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \\ 25.a. B \begin{bmatrix} 0.35 & 0.65 & 0 \end{bmatrix} \\ C \begin{bmatrix} 0.45 & 0 & 0.55 \end{bmatrix} \end{matrix}$$

b. A: 51%; B: 32.5%; C: 16.5% c. A: 69.8%;
B: 21.1%; C: 9.1%

26.a,b.



c. $T = \begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$, $|T| = 5$; $\frac{\text{area } \triangle X'Y'Z'}{\text{area } \triangle XYZ} = 5$ and
the \triangle have the same orientation.

Cumulative Review, pages 562-563

1.a. P



b. 414.8 km; 185.7°

2.a. $(-6, 8)$ b. $(3, 1)$ c. $5\sqrt{2}$

3.a. $(x, y) = (-2, -4) + t(-2, 5)$

b. $x = -2 - 2t$, $y = -4 + 5t$

4.a. Vel. = $(2, -4)$, speed = $2\sqrt{5}$

b. $t = 1$; $(3, -5)$

5. 62.3° 6. $(x, y, z) = (2, 0, 5) + t(-5, -1, -1)$;

$x = 2 - 5t$, $y = -t$, $z = 5 - t$ 7. $2x + y - z = 0$

8. 336 9. $(3, 2)$ 10. $(6, 9, -1)$;

$6x + 9y - z = 1$ 11.a. Geom.; $t_n = -\frac{4}{9}\left(-\frac{3}{2}\right)^n$

b. Neither; $t_n = \frac{2n}{2n+1}$ c. Arith.; $t_n = n - \frac{1}{3}$

12.a. $-2, 0, 4, 12, 28$; b. $t_n = 2^n - 4$ c. 1020

13. -180 14. $\frac{364}{3}$ 15.a. 1 b. Does not exist.

16.a. 3 b. ∞ c. Does not exist. 17. $-4 < x < 4$;

$\frac{8}{x+4}$ 18. $\frac{115}{333}$ 19. $\sum_{n=1}^{20} (5n+2)$; 1090

20. For $n = 1$, $\frac{1}{2(1)+1} = \frac{1}{3} = \frac{1}{1 \cdot 3}$; if

$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}$, then

$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2k-1)(2k+1)} +$

$\frac{1}{(2(k+1)-1)(2(k+1)+1)} =$

$\frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)} =$

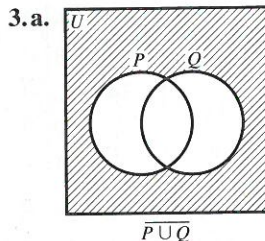
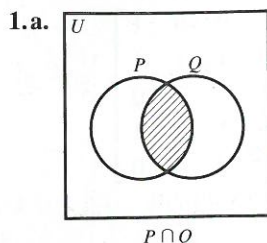
$\frac{k(2k+3)}{(2k+1)(2k+3)} + \frac{1}{(2k+1)(2k+3)} =$

$\frac{2k^2+3k+1}{(2k+1)(2k+3)} = \frac{(2k+1)(k+1)}{(2k+1)(2k+3)} = \frac{k+1}{2k+3}$.

Class Exercises, page 568 1. The set of freshmen
music majors; 15 2. The set of students who are
either freshmen or music majors; 350 3. The set
of students who are not freshmen; 700 4. The set
of students who are not music majors; 935 5. The
set of music majors who are not freshmen; 50

6. The set of freshmen who are not music majors; 285 7. The set of students who are neither freshmen nor music majors; 650 8. The set of students who are either freshmen or are not music majors; 950 9.a. $B \cap \bar{A}$ b. $A \cap B$ c. $\bar{A} \cup \bar{B}$ or $\bar{A} \cap \bar{B}$ 10.a. A b. \emptyset c. U d. \emptyset e. U f. A

Written Exercises, pages 568–571



5.a. Teachers of either math or physics b. Teachers of math but not of physics 7.a. Teachers of biology or of both physics and chemistry b. Teachers of both biology and chemistry 9.a. Teachers of neither math nor physics nor chemistry b. Same as part a. 11. 28 13. 12 19.a. 11 b. 12 21. \emptyset 25.b. $\overline{A \cap B} = \bar{A} \cup \bar{B}$ c. $\bar{A} \cup (\bar{B} \cap \bar{C})$ 27. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(B \cap C) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

Class Exercises, page 574 1.a. 2 b. 6 c. 24
2. 3,628,800 3. 608,400 4.a. 60 b. 180 5.a. 8
b. 6 6. 720 7.a. 362,879 b. 1,133,836,704,000

Written Exercises, pages 575–577 1.a. 120 b. 720
c. 5040 d. 1 3. 120 5. 1024 7.a. 17,576
b. 13,800 9. 11,880 11.b. 14 branches, 8 leaves
c. 2046 branches, 1024 leaves 13.a. 362,880
b. 40,320 c. 5040 15. 156 17. 2 digits:
1,757,600; 3 digits: 17,576,000; total: 19,333,600
19.a. 648 b. 252 21. 542 23.a. 160 b. 640
c. 9999 d. 6,399,360 e. 1,023,897,600
25.a. 362,880 b. 30,240 27. $26^2 = 676 < 677$
31. $\log_{10} 9! \approx 5.56$; $\log_{10} 10! \approx 6.56$
33.a. $\frac{10!}{6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6!} = 10 \cdot 9 \cdot 8 \cdot 7$

Class Exercises, page 580 1.a. 20 b. 10
2.a. 120 b. 20 3.a. 720 b. 120 4.a. 24 b. 1
5. 720 6. 120 7. The order of the three nos. is important.

8.a. ABC ACB BAC BCA CAB CBA
ABD ADB BAD BDA DAB DBA
ACD ADC CAD CDA DAC DCA
BCD BDC CBD CDB DBC DCB

b. A, B, and C: 6; A, C, and D: 6 c. ABC, ABD, ACD, BCD

Written Exercises, pages 580–582 1.a. 380 b. 190
3.a. 210 b. 5040 5.a. 1,313,400 b. 7,880,400
7.a. 336 b. 56 9.a. 665,280 b. 924 11.a. 35
b. 35 13.a. ${}_{100}C_2 = {}_{100}C_{98} = \frac{100!}{2!98!}$ 15.a. 1
b. 1 17. 22,764 19. 48 21.a. 792 b. 658,008
c. 1,940,952 23.a. 15 b. There will be fewer line segs. if three or more pts. are collinear. 25. 10
29. 5.36×10^{28}

Class Exercises, page 585 1.a. 24 b. 12
2.a. 120 b. 60 3.a. 720 b. 180 4.a. 39,916,800
b. 1,663,200 7. 84 8. BCDAE

Written Exercises, pages 585–587 1. 10,080
3. 34,650 5. 840 7. 9.71×10^{14} years
9.a. 3003 b. 21 c. 35 d. 735 11. 24
13. 1.37×10^{11} 15.b. 8.25×10^{12} 17. 6188

Mixed Combinatorics Exercises, pages 587–589
1. 161,700 3.a. 6561 b. 5832 c. 2000 5.a. 870
b. 435 7. 60 ways from A to D; 3600 round trips
9. 83 11.a. 1 min b. 13 h 13.a. 70 b. 35 c. 35
15. 15 17. 64 19.a. 86,400 b. 28,800
21. 37,512

Class Exercises, page 592
2. 1 7 21 35 35 21 7 1 3.a. 1 8 28 56
b. $x^8, 8x^7y, 28x^6y^2, 56x^5y^3$
c. $x^8, -8x^7y, 28x^6y^2, -56x^5y^3$
4. $a^3 - 3a^2b + 3ab^2 - b^3$
5. $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
6. $a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$ 7.a. $15x^4y^2$
b. $84x^6y^3$

Written Exercises, pages 592–594
1.a. $a^3 + 3a^2b + 3ab^2 + b^3$
b. $8000 + 1200 + 60 + 1 = 9261$
c. $8000 - 1200 + 60 - 1 = 6859$
3.a. $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$
b. $a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$
c. $32a^5 + 80a^4 + 80a^3 + 40a^2 + 10a + 1$
5.a. $x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$ b. $x^7 - 7x^6y + 21x^5y^2 - 35x^4y^3 + 35x^3y^4 - 21x^2y^5 + 7xy^6 - y^7$