

Pre-Calculus
Review -- Ch 15.5

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Period _____ Date _____

- The polynomial $a+b$ is called a binomial because it has two terms.
- Consecutive powers of $a+b$ [e.g., $(a+b)^1, (a+b)^2, (a+b)^3$, etc.] are referred to collectively as the binomial Expansion.
- The binomial Theorem tells us that for any positive integer n

$$(a+b)^n = {}_n C_0 a^n b^0 + {}_n C_1 a^{n-1} b^1 + {}_n C_2 a^{n-2} b^2 + \dots + {}_n C_n a^0 b^n$$
- ${}_n C_r = \frac{n!}{r!(n-r)!}$ in factorial notation.
- For any positive integer n , ${}_n C_0 = 1$; ${}_n C_1 = n$; ${}_n C_n = 1$
- $a^0 = 1$; $b^1 = b$; $a^m a^n = a^{m+n}$; $\frac{a^m}{a^n} = a^{m-n}$; $(a^m)^n = a^{mn}$; $(-1)^n = 1$ if n even
 $(-1)^n = -1$ if n odd
- Complete Pascal's Triangle through $n = 10$:

n	Coefficients of $(a+b)^n$
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1
7	1 7 21 35 35 21 7 1
8	1 8 28 56 70 56 28 8 1
9	1 9 36 84 126 126 84 36 9 1
10	1 10 45 120 210 252 210 120 45 10 1

- For the expansion of $(a+b)^n$ where $n \geq 5$, provide the following:

- The coefficient of the 5th term is ~~_____~~ ${}_n C_5$
- The exponent of a in the 5th term is 5
- The exponent of b in the 5th term is $n-5$
- For any term, the sum of the exponents of a and b is always n
- The entire 5th term is $({}_n C_5) a^5 b^{(n-5)}$

9. Expand each of the following:

a. $(x^2 - y^2)^3 = {}_3C_0(x^2)^3(-y)^0 + {}_3C_1(x^2)^2(-y)^1 + {}_3C_2(x^2)(-y)^2 + {}_3C_3(x^2)^0(-y)^3$
 $1 \cdot x^6 + 3x^4(-y) + 3x^2y^2 + 1(-y^3)$
 $x^6 - 3x^4y + 3x^2y^2 - y^3$

b. $(2x^2 - 1)^4 = {}_4C_0(2x^2)^4(-1)^0 + {}_4C_1(2x^2)^3(-1)^1 + {}_4C_2(2x^2)^2(-1)^2 + {}_4C_3(2x^2)(-1)^3 + {}_4C_4(1)(-1)^4$
 $1(16x^8) + 4(8x^6)(-1) + 6(4x^4)(1) + 4(2x^2)(-1) + 1(1)$
 $16x^8 - 32x^6 + 24x^4 + 8x^2 + 1$

c. $(x^2 + 2)^3 = {}_3C_0(x^2)^3(2)^0 + {}_3C_1(x^2)^2(2)^1 + {}_3C_2(x^2)(2)^2 + {}_3C_3(x^2)^0(2)^3$
 $1 \cdot x^6 + 3(x^4)2 + 3(x^2)4 + 8 = x^6 + 6x^4 + 12x^2 + 8$

10. In the expansion of $(a+b)^{15}$, find

a. The 9th term ${}_{15}C_9 a^9 b^6 = 5005 a^9 b^6$

b. The 10th term ${}_{15}C_{10} a^{10} b^5 = 3003 a^{10} b^5$

11. In the expansion of $(a-b)^{15}$, find:

a. The 9th term ${}_{15}C_9 (a)^9 (-b)^6 = 5005 a^9 b^6$

b. The 10th term ${}_{15}C_{10} (a)^{10} (-b)^5 = -3003 a^{10} b^5$

12. Find the 8th term of $(p+q)^{50}$ ${}_{50}C_8 (p)^8 (q)^{42} = 536878650 p^8 q^{42}$

13. Find the coefficient of x^2 in the expansion of $(x - \frac{1}{x})^{10}$ $\frac{{}_{10}C_2 (x)^2 (-\frac{1}{x})^8}{45 x^2 (\frac{1}{x^2})} = \frac{45}{x^6}$

14. Find the coefficient of x^{10} in the expansion of $(x^2 - 1)^9$ ${}_{10}C_5 (x^2)^5 (-1)^4 = 252 x^{10}$