Linear Inequalities

1) An inequality is different from an equation by the sign that lies in between the two sides. Instead of an "=" there will be:

$\leq$	meaning	"less than or equal to"
$\geq$	meaning	"greater than or equal to"
$\square$	meaning	"less than"
$\square$	meaning	"greater than"

The biggest conceptual difference between an equation and an inequality is:

An equation will have a **finite** number of solutions

An inequality will have an **infinite** number of solutions

Solutions of an inequality:

- With an inequality, you are finding all values of x for which the inequality is true.
- Such values are solutions and are said to satisfy the inequality.
- The set of all real numbers that are solutions of an inequality is the solution set of the inequality.
- The set of all points on the real number line that represent the solution set is the graph of the inequality.

Write an inequality to represent each interval and state whether the interval is bounded or unbounded.

$x \in (-3,5]$	$-3 < x \leq 5$	Bounded / Unbounded
$x \in (-3,\infty)$	$x > -3$	Bounded / Unbounded
$x \in [0,2]$	$0 \leq x \leq 2$	Bounded / Unbounded
$x \in (-\infty,\infty)$	$x \in \mathbb{R}$	Bounded / Unbounded

2) There are properties of inequalities we can use. In your groups you need to decide if when you start with a true inequality, will adding, subtracting, multiplying and dividing both sides by the same number does the inequality stay true.

a) Start with a true inequality 2 < 3

b) Pick your favorite positive number 10

c) Pick your favorite negative number -2

d) Take your inequality from part (a) and add the number from part (b) to both sides. Is your new inequality true? Yes

e) Take your inequality from part (a) and add the number from part (c) to both sides. Is your new inequality true? Yes

f) Take your inequality from part (a) and multiply the number from part (b) to both sides. Is your new inequality true? Yes

g) Take your inequality from part (a) and multiply the number from part (c) to both sides. Is your new inequality true? No

h) Fill in the inequality with your numbers from part (b) and (c) -2 < 10

i) Add the left sides of the inequalities from part (a) and part (h), then add the right sides of the inequalities from part (a) and part (h). What inequality is true?

$0 < 13$

Yes

Ok, so now fill in the following properties given your results from the previous problems:

Let a, b, c, and d be real numbers

Addition of a Constant Property of Inequalities – If $a < b$, then $a + c$ $< b + c$

Addition of Inequalities Property - If $a < b$ and $c < d$, then $a + c$ $< b + d$

Multiplication of a Constant Property of Inequalities

If $a < b$ and $c > 0$, then $a \cdot c$ $< bc$

If $a < b$ and $c < 0$, then $a \cdot c$ $> bc$

If $a < b$ and $c = 0$, then $a \cdot c$ $= b \cdot c$

The last property we want to talk about is the Transitive Property.

Reminder from Geometry The Transitive Property of Equality states:

If $a = b$ and $b = c$, then $a = c$.

Use any examples you want and figure out and construct the Transitive Property of Inequalities.

If $a < b$ and $b < c$, then $a < c$

3) Using these properties, solve the following inequalities.

(By solve I mean find all of the values of x that would make the inequality true)

a) $5x - 7 > 3x + 9$

$2x > 16$

$x > 8$

b) $1 - \frac{3x}{2} \geq x - 4$

$5 \geq \frac{5x}{2}$

$10 \geq 5x$

$2 \geq x$

c) $-3 \leq 6x - 1 < 3$

$-2 \leq 6x < 4$

$-1/3 \leq x < 2/3$

4) Absolute Value Inequalities: Just like with absolute value equations, you are going to ultimately have to solve 2 inequalities. The first equation you solved when it was an equation just took the inside of the absolute value expression and set it equal to the other side. (You're going to do the same thing with inequalities for this one). The second equation you made, took the inside of the absolute value expression and set it equal to the opposite of the other side (or the other side multiplied by -1). You're going to do a VERY similar thing, but (here's the tricky part) when I multiply an inequality by a -1 what do you HAVE to do with the inequality sign? Flip it

Try a few examples:

a) $|x - 5| < 2$

$-2 < x - 5 < 2$

$\boxed{3 < x < 7}$

b) $|x + 3| \geq 7$

$x + 3 \geq 7$

$x \geq 4$

$x + 3 \leq -7$

$x \leq -10$

Send up a representative for your group with this station completed. I will check your work and give you the next portion.

Applications of Linear Inequalities

Now you are going to apply these concepts to solving application problems. These application problems are VERY similar to the application problems we have previously been working on.

Steps

1- Draw a Diagram if Applicable or Think about the situation
2- Set up an inequality
3- Solve the inequality
4- Check the solutions to see if they make sense

Try some ☺

1) Your department sends its copying to the photocopy center of your company. The center bills your department \$0.10 per page. You have investigated the possibility of buying a departmental copier for \$3000. With your own copier, the cost per page would be \$0.03. The expected life of the copier is 4 years. How many copies must you make in the 4-year period to justify buying the copier?

c - copies

$$0.1c \geq 3000 + 0.03c$$

$$\frac{0.07c}{0.07} \geq \frac{3000}{0.07}$$

$$c \geq 42857$$

2) In order for an investment of \$750 to grow to more than \$825 in 2 years, what must the annual interest rate be?

$[A = P(1+rt)]$

$$\frac{825}{750} < \frac{750(1+2r)}{750}$$

$$1.1 < 1 + 2r$$

$$\frac{0.1}{2} < \frac{2r}{2}$$

$$0.05 < r$$

3) The average salary S (in thousands of dollars) for elementary and secondary teachers in the United States from 1987 to 1997 is approximated by the model $S = 30{,}944 + 1{,}187t$, when $-3 \le t \le 7$, where t = -3 represents 1987. According to this model, when will the average salary exceed $42,000?

$30{,}944 + 1{,}187t > 42000$

$1{,}187t > 11056$

$t > 9.3$

year 2000

4) The side of a square is measured as 10.4 inches with a possible error of $\frac{1}{16}$ inch. Using these measurements, determine the interval containing the possible areas of the square.

$10.3375 \le S \le 10.4625$

$106.864 \le S^2 \le 109.464$

5) You buy a bag of oranges for $0.95 per pound. The weight that is listed on the bag is 4.65 pounds. The scale that weighed the bag is accurate to within 1 ounce. How much might you have been undercharged or overcharged?

$4.55 < 4.65 < 4.75$

$\times 0.95 \quad \times 0.95 \quad \times 0.95$

$4.3225 < 4.4175 < 4.5125$

$-4.4175 \quad -4.4175$

$-0.095 < 0 < 0.095$

$-0.10 < x < 0.10$

Send up a different representative for your group with this station completed. I will check your work and give you the next portion.

Polynomial Inequalities

Try this application problem now…. It's a little different.

JP is standing on a bridge that's 12 feet above the ground. He throws a ball up in the air with an initial velocity of 25 ft/s.

$$s = -16t^2 + v_0 t + s_0$$

a) When will the ball be above the bridge? $s > 12$

$0 < t < 1.56$

b) When will the ball be below the bridge? $s < 12$

$t > 1.56$

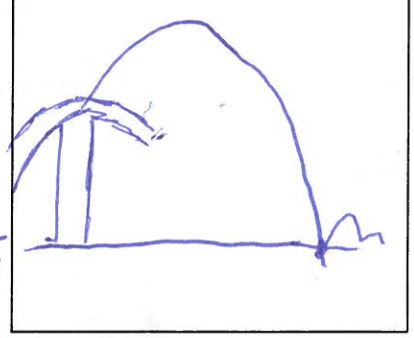
c) When will the ball be more than 20 feet in the air? $s > 20$

$0.449 < t < 1.11$

d) When will the ball be at most 15 feet above the ground? $s > 15$

$0 < t < 1.31$ or $t > 1.43$

Diagram



This was an example of a quadratic inequality. Polynomial inequalities are a little more difficult than linear inequalities were, but if you follow these steps, you can solve any one thrown your way!

Solving Polynomial Inequalities

- To solve a polynomial inequality, you can use the fact that a polynomial can change signs only at its zeros (the x values that make the polynomial equal to zero)
- Between two consecutive zeros, a polynomial must be entirely positive or negative.
- When the real zeros of a polynomial are put in order, they divide the real number line into intervals in which the polynomial has no sign changes.
- These zeros are the critical number of the inequality, and the resulting intervals are the test intervals for the inequality

To determine the intervals on which the values of a polynomial are entirely negative or positive, use the following steps:

- Find all real zeros of the polynomial, and arrange the zeros in increasing order (from smallest to largest). These zeros are the critical numbers of the polynomial.
- Use the critical numbers of the polynomial to determine its test intervals.
- Choose one representative x value in each test interval and evaluate the polynomial at that value. If the value of the polynomial is negative, the polynomial will have negative values for every value in the interval. If the value of the polynomial is positive, the polynomial will have positive values for every x value in the interval.

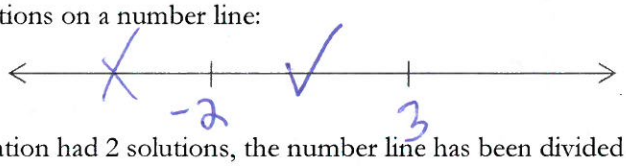
1) Let's fill in this example below to help solve $x^2 - x - 6 < 0$

a) Change it to an equation $x^2 - x - 6 = 0$

b) Solve the equation: $(x-3)(x+2) = 0$

x = -2 and x = 3

c) Plot your solutions on a number line:



d) Since our equation had 2 solutions, the number line has been divided into 3 test intervals. Pick values that lie in each of the intervals:

1st interval test number: -3 2nd interval test number: 0 3rd interval test number: 4

e) Plug in each of the numbers from part (d) into the original inequality. Decide if that number makes the inequality true or false.

1st interval: **T / F** 2nd interval: **T / F** 3rd interval: **T / F**

f) If the test number makes the inequality true, ALL of the numbers on that interval will be make the inequality true. Knowing this what are your solutions to the inequality?

$-2 < x < 3$

2) Try some using the same process we did in #1

a) $x^2 + 2x \geq 15$

$x + 2x - 15 \geq 0$

$(x+5)(x-3) \geq 0$

✓ X ✓

-5 3

$x \leq -5$ or $x \geq 3$

b) $2x^3 - 3x^2 - 32x > -48$

$x^2(2x-3) - 16(2x-3) > 0$

$(x-4)(x+4)(2x-3) > 0$

X ✓ X ✓

-4 3/2 4

$-4 < x < 3/2$ or $x > 4$

c) $x^2 + 2x + 4 \geq 0$

$-2 \pm \sqrt{2^2 - 4(1)(4)}$

No Sol

$1^2 + 2(1) + 4 \geq 0$

True

$x \in \mathbb{R}$

d) $x^2 + 2x + 1 \leq 0$

$(x+1)(x+1) \leq 0$

X ● X

-1

$x = -1$

e) $x^2 + 3x + 5 < 0$

$3^2 - 4(1)(5) < 0$

$b^2 - 4ac$

$1^2 + 3x + 5 > 0$

No Sol

f) $x^2 - 4x + 4 > 0$

$(x-2)(x-2) > 0$

✓ X ✓

2

$x \neq 2$

3) Use the same process to answer this question:

Find the domain of the expression $\sqrt{64 - 4x^2}$

$64 - 4x^2 \geq 0$

$(8 - 2x)(8 + 2x) \geq 0$

X ✓ X

-4 +4

$-4 \leq x \leq 4$

Send up a different representative for your group with this station completed. I will check your work and give you the next portion.

Rational Inequalities

1) What if you are dealing with inequalities that are made up of rational expressions? It's not terribly different than the polynomial inequalities. Let's look at one together.

$$\frac{1}{x} - x > 0$$

a) Change it to an equation. $\frac{1}{x} - x = 0$

b) Take the side of the equation with the rational expressions and combine them so there is only one rational expression on that side of the equation.

$$\frac{1-x^2}{x} = 0$$

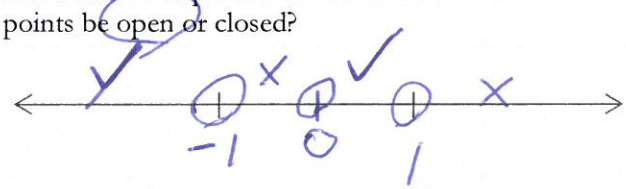
c) Ok let's think about division for a second. In order to get zero as the quotient, what must be true?

$x = 1, -1$

d) Therefore the only thing we really care about is the Numerator being equal to zero. So what is the equation we actually want to solve now? Set it up and do it!

$1 - x^2 = 0$

e) These are now the values that will be plotted on our number line. Think about this, looking at your inequality symbol should these points be open or closed?



f) Here's where rational inequalities get a little different. What do you think we also need to think about, since there is an x in the denominator? Domain / Denominator $\neq 0$

g) Plot your domain restrictions on your number line as additional critical values. Just a thought… should these be open or closed circles on the number line? Add this to the number line in part (e).

h) Find the intervals that make the inequality true. Since the inequality has 0 on the one side, really you only care if it is positive or negative not what the actual number is, which makes your life MUCH easier.

i) What are the solutions to this inequality? Keep in mind the domain of the original problem.

$x < -1, \; 0 < x < 1$

Note…you want always set the inequality equal to zero before doing anything else if it's not zero to begin with.

2) Try some on your own.

a) $\frac{1}{x} - 4 < 0$

$\frac{1-4x}{x} < 0$

$x < 0$ or $x > 1/4$

b) $\frac{x+12}{x+2} - 3 \geq 0$

$x+12-3x-6$

$\frac{-2x+6}{x+2} \geq 0$

$-2 < x \leq 3$

c) $\frac{5+7x}{1+2x} < 4$

$5+7x-4(1+2x)$

$5+7x-4-8x$

$\frac{1-x}{1+2x} < 0$

$x < -1/2$ or $x > 1$

d) $\frac{5}{x-6} > \frac{3}{x+2}$

$5x+10-3x+18$

$\frac{5(x+2)-3(x-6)}{(x-6)(x+2)} > 0$

$\frac{2x+28}{(x-6)(x+2)} > 0$

$-14 < x < -2$ or $x > 6$

e) $\frac{1}{x} \geq \frac{1}{x+3}$

$\frac{1(x+3)-1x}{x(x+3)} \geq 0$

$\frac{3}{x(x+3)} \geq 0$

$x < -3$ or $x > 0$

f) $\frac{x^2+x-6}{x} \geq 0$

$\frac{(x+3)(x-2)}{x} \geq 0$

$-3 \leq x < 0$ or $x \geq 2$

g) $\frac{3x}{x-1} \leq \frac{x}{x+4}$

$\frac{3x(x+4)-x(x-1)}{(x-1)(x+4)} \leq 0$

$\frac{3x^2+12x-x^2+x}{(x-1)(x+4)} \leq 0$

$\frac{x(2x+13)}{(x-1)(x+4)} \leq 0$

$-\frac{13}{2} \leq x < -4$ or $0 \leq x < 1$

Send up a different representative for your group with this station completed. I will check your work and give you the FINAL portion. ☺

More Application Problems

Let's just try some ☺

1) A projectile is fired straight upward from ground level with an initial velocity of 128 feet per second.

a) At what instant will it be back at ground level?

$s = -16t^2 + 128t = 0$

$-16t(t-8) = 0$

$t = 8$

b) When will the height be less than 128 feet?

$-16t^2 + 128t < 128$

$-16t^2 + 128t - 128 < 0$

$-16(t^2 + 8t - 8) < 0$

$-16(t - 1.17)(t - 6.83) < 0$

$t < 1.17$ or $t > 6.83$

2) A rectangular parking lot with a perimeter of 440 feet is to have an area of at least 8000 square feet. Within what bounds must the length of the rectangle lie?

$440 = 2w + 2l$

$220 = w + l$

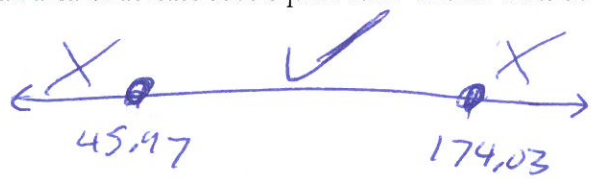
$w = 220 - l$

$(220 - l)(l) \geq 8000$

$-l^2 + 220l - 8000 \geq 0$

$-(l - 220l + 8000) \geq 0$

$(t - 45.97)(t - 174.03) \leq 0$



$45.97 \leq X \leq 174.03$

3) P dollars, invested at interest rate r compounded annually, increases to an amount $A = P(1+r)^2$ in 2 years. If an investment of \$1000 is to increase to an amount greater than \$1100 in 2 years, then the interest rate must be greater than what percent?

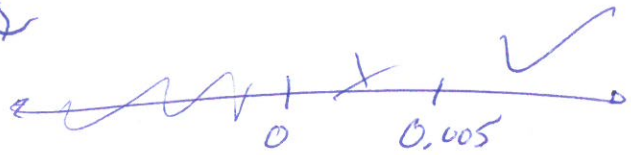
$1000(1+r)^2 > 1100$

$(1+r)^2 - 1.1 > 0$

$1 + 2r + r^2 - 1.1 > 0$

$r^2 + 2r - 0.1 > 0$

$(r - 0.005)(r + 2) > 0$



$r > 0.005$

4) The revenue and cost equations for a product are $R = x(50 - 0.0002x)$ and $C = 12x + 150,000$, where R and C are measured in dollars and x represents the number of units sold. How many units must be sold to obtain a profit of at least \$1,650,000?

$R - C = \text{Profit}$

$x(50 - 0.0002x) - (12x + 150000) > 1,650,000$

$-0.0002x^2 + 38x - 150,000 > 0$
$-1650,000$

$-0.0002x^2 + 38x - 1,800,000 > 0$ $\quad 90,000 \le x \le 100,000$

$(x - 90000)(x - 100,000) > 0$

90,000 100,000

5) The receipts from Broadway shows (in millions of dollars) from 2000 to 2006 can be modeled by $R = 4.7t^2 + 277$ where t is the time, with $t = 0$ corresponding to 2000. According to this model, when will the receipts from Broadway shows exceed \$1 billion?

$4.7t^2 + 277 > 1000,000,000$

$4.7t^2 - 999999,723 > 0$

$t = \pm 14586.5$

$\frac{-14586.5}{1000}$ $\quad$ $\frac{14586.5}{1000}$

2015

Send up a different representative for your group with this station completed. I will check your work and you can get started on your HW!!