Factoring Review for Algebra II

The saddest thing about not doing well in Algebra II is that almost any math teacher can tell you going into it what's going to trip you up. One of the first things they'll say while shaking their head and chuckling sadly is "factoring." Factoring is so fundamental to success in Algebra II that to not be able to factor is pretty much to not be able to pass the course. We're going to try to put you on the right path in these next few pages.

Can You Distribute?

Factoring is actually the inverse operation of distributing. Another, more awesome name for factoring might have been "anti-distributing." So to be really, really good at factoring you first need to be really, really, hardly-even-need-to-think-about-it good at distributing—and there's a good chance you're not.

So, here's what the distributive property says: $(a)(b+c)=ab+ac$, but it can also be written as $(b+c)(a)=ba+ca$. Let's use the distributive property on a couple of examples. A common way for problems involving the distributive property to be asked is for you to be told to "expand" whatever you're given.

Example: Expand $3(x+y)$.

Work: $3x+3y$. There's not really much work to show there.

Example: Expand $x(x+2)$.

Work: $x(x)+x(2)=x^2+2x$

Example: Expand $y(x+3)$.

Work: $y(x)+y(3)=xy+3y$. I rewrote this at the end because you usually put variables in alphabetical order in your final answer.

That last example isn't any different from the previous examples, but I was trying to set you up to do something a little more complicated. I'm going to replace the y in the previous problem with $x-5$ in the next example. Keep in mind what we did with $y(x+3)$ as you work through this (or read through my work).

At this point if you'd rather watch me do some examples, click here.

Example: Expand $(x-5)(x+3)$.

Work:

$$(x-5)(x)+(x-5)(3)=x(x)-5(x)+3(x)-5(3)$$
$$=x^2-5x+3x-15=x^2-2x-15$$

Example: Expand $(3x+2)(2x-5)$.

Work:

$$(3x+2)(2x)+(3x+2)(-5)=3x(2x)+2(2x)+3x(-5)+2(-5)$$
$$=6x^2+4x-15x-10=6x^2-11x-10$$

These last two are probably the most important examples so far. You really need to be able to expand these sorts of things for a few reasons. First, to go from the answer of those examples back to the original problem is usually what we mean by factoring. Second, expanding your answer is how you check your work.

If you get confused about what's going on, replace one of the factors with *y*, distribute the *y*, then go back to the original factor and keep going. I'll do that with the next example to demonstrate what I mean.

Example: Expand $(4x-5)(7x+3)$

Work: Let $y=4x-5$ and the problem is now to expand $y(7x+3)$, which gives

$$y(7x)+y(3)$$

Now, replace the *y* with the original $4x-5$ and you have,

$$(4x-5)(7x)+(4x-5)(3)$$

So we can finish the problem from there,

$$(4x-5)(7x)+(4x-5)(3)=4x(7x)-5(7x)+4x(3)-5(3)$$
$$=28x^2-35x+12x-15=28x^2-23x-15$$

Okay, two more examples of this, then we'll get to the point. But trust me, if you can't do this, you're not going to be able to factor.

Example: Expand $(5x-4)(3x-2)$.

Work: I don't always (in fact, I never personally do it) replace one of the factors with a *y* before doing the problem, but there's nothing wrong with doing that. Here's how I would actually solve this problem.

$$(5x-4)(3x)+(5x-4)(-2)=5x(3x)-4(3x)+5x(-2)-4(-2)$$
$$=15x^2-12x-10x+8=15x^2-22x+8$$

Remember, if you get confused about how to distribute, just let *y* replace one of the factors, do the problem, then switch it back and finish. It makes the first part look significantly easier.

Example: Expand $(3x-7)(9x+5)$.

Work:

$$(3x-7)(9x)+(3x-7)(5)=3x(9x)-7(9x)+3x(5)-7(5)$$
$$=27x^2-63x+15x-35=27x^2-48x-35$$

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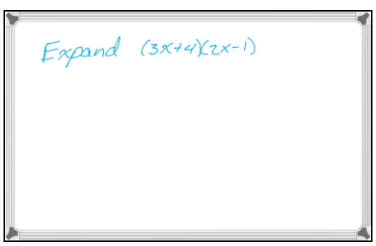

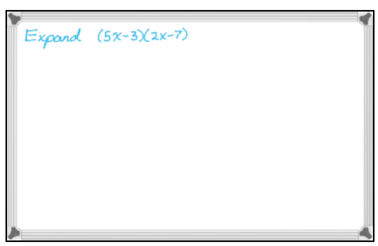

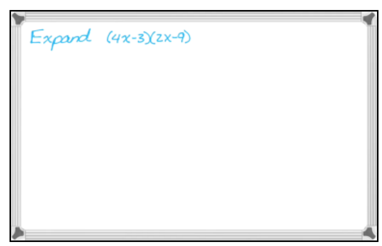

Now go back and work along with the rest of the examples! Click here to do that.

Now Do This

Expand each of the following.

1. $7x(5x^2+3x+4)$ (Sol.)
2. $5x^2(3x+2)$ (Sol.)
3. $5x(x^2-11x+8)$ (Sol.)
4. $(7x+3)(2x-5)$ (Sol.)
5. $(5x-3)(3x-2)$ (Sol.)
6. $(4x+9)(2x-11)$ (Sol.)

Greatest Common Factoring

The most basic form of factoring is taking out the greatest common factor (GCF) of all of the terms in the expression. An expression is a collection of terms. Things like $5x^2 + 3x - 5$ and $9x + 3$ are examples of expressions. A term is a single part of the expression. The terms are the things separated by plus and minus signs, but they keep the sign to their left. So the terms of $5x^2 + 3x - 5$ are $5x^2$, $3x$, and -5.

Example: Identify the terms of the expression $-20x^3 + 16x^2 - 4x + 8$.

Work: There are four terms. They are $-20x^3$, $16x^2$, $-4x$, and 8.

Did you notice that all of the terms in that expression are multiples of 4? They are. That means there's a greatest common factor or GCF. The first step in factoring any expression is to factor out the GCF. To figure out what you're left with when you factor the GCF out of a term, you divide the term by the GCF. I'll demonstrate.

Example: Factor out the GCF of the expression $-20x^3 + 16x^2 - 4x + 8$.

Work: We need to divide each of the terms of the expression by 4.

$\frac{-20x^3}{4} = -5x^3$, $\frac{16x^2}{4} = 4x^2$, $\frac{-4x}{4} = -x$, and $\frac{8}{4} = 2$. So we end up with our factor expression: $-20x^3 + 16x^2 - 4x + 8 = 4\left(-5x^3 + 4x^2 - x + 2\right)$.

You can always check your answer by using the distributive property.

Of course, you're probably thinking you could have done that work in your head. That's fine. Almost everyone does. But I wanted you to see that there is actually work that you *could* do—and sometimes you might find that you have to do the work so it's good to know it's possible.

Example: Factor out the GCF of $24x - 32$.

Work: Since 8 is the largest common factor of 24 and 32 it is the GCF.

$$24x - 32 = 8(3x - 4)$$

Remember to check by distributing!

Example: Factor out the GCF of $50x^3 + 25x^2 - 100x$.

Work: Since 25 is the largest common factor of 50, 25, and 100 and x is the highest common power of x, x^2, and x^3, the GCF is $25x$.

$$50x^3 + 25x^2 - 100x = 25x\left(2x^2 + x - 4\right)$$

Remember to check by distributing!

Example: Factor out the GCF of $9x^4 + 12x^3 - 15x^2$.

Work: The greatest common factor of all three terms is $3x^2$.

$$9x^4 + 12x^3 - 15x^2 = 3x^2\left(3x^2 + 4x - 5\right)$$

Remember to check by distributing!

Factoring out the GCF is a really important skill in part because it's pretty much going to be used three times on every problem you do using the next method we review. First a little practice for you.

Now Do This

1. $36x - 27$ (Sol.)
2. $5x^2 + 35x - 45$ (Sol.)
3. $18x^3 - 3x^2 + 6x$ (Sol.)
4. $4x^4 + 12x^3 - 16x^2 + 80x$ (Sol.)
5. $9x^3y^2 - 6x^2y^3$ (Sol.)
6. $12x^4y^3 + 24x^2y^4 + 60xy^4$ (Sol.)

Factoring by Grouping

Factoring by grouping is a kind of neat way of factoring in which instead of finding an overall GCF to begin with you break up the terms into a couple of groups, find the GCF of each group and factor it out, then take a look at what remains. Sometimes what remains is a new expression that has yet another GCF that was hidden at the outset. As with most things, it's much easier to do than it is to explain.

Let's try to factor $6x^2-2x+9x-3$. You can see that there is no GCF for all four terms other than 1, which is a pretty worthless GCF. So, let's reorganize,

$$6x^2-2x+9x-3=\left(6x^2-2x\right)+\left(9x-3\right)$$

Now if you look at each pair of terms in parentheses we can find a different GCF for each of them.

$$\left(6x^2-2x\right)+\left(9x-3\right)=2x\left(3x-1\right)+3\left(3x-1\right)$$

Now that we've done this, the two things we're left with actually do have a GCF! In this case the GCF is $3x-1$. You can see it right there, it's obviously a factor of both terms. So we finish this off by factoring out this new, formerly hidden GCF.

$$2x\left(3x-1\right)+3\left(3x-1\right)=\left(3x-1\right)\left(2x+3\right)$$

And that's how it works. We actually factored out three GCFs along the way. First we grouped the terms into two groups and took the GCF out of each group—that was two of the GCFs we factored out. Then we looked at what we were left with and factored out the GCF of that—that's the third time we factored out a GCF. This will work often and you'll be doing it a lot. Let's practice.

If you'd rather watch me do a few problems first, click here.

Example: $10x^2-25x+4x-10$

Work:

$$10x^2-25x+4x-10=\left(10x^2-25x\right)+\left(4x-10\right)$$
$$=5x\left(2x-5\right)+2\left(2x-5\right)=\left(2x-5\right)\left(5x+2\right)$$

For the next example I'm going to do basically the same problem over again, but in this case the two middle terms will be switched. It's always really amazed me how this method works regardless of what order the middle terms take.

Example: $10x^2 + 4x - 25x - 10$

Work:

$$10x^2 + 4x - 25x - 10 = \left(10x^2 + 4x\right) + \left(-25x - 10\right)$$
$$= 2x\left(5x + 2\right) - 5\left(5x + 2\right) = \left(5x + 2\right)\left(2x - 5\right)$$

If you go back and look at the last two examples you'll see that we got the same answer both times. The only difference was the GCFs that we took out along the way. In the end, though, we've ended up in the same place. It's amazing. You also might notice—but maybe not without me suggesting that you notice it—that no matter how you write the middle term, the original thing we factored simplifies down to $10x^2 - 21x - 10$. Maybe that suggests a way we can factor trinomials if we could figure out how to break them up in this way.

Example: $8x^2 + 28x - 6x - 21$

Work:

$$8x^2 + 28x - 6x - 21 = \left(8x^2 + 28x\right) + \left(-6x - 21\right)$$
$$= 4x\left(2x + 7\right) - 3\left(2x + 7\right) = \left(2x + 7\right)\left(4x - 3\right)$$

Example: $63x^2 + 35x - 36x - 20$

Work:

$$63x^2 + 35x - 36x - 20 = \left(63x^2 + 35x\right) + \left(-36x - 20\right)$$
$$= 7x\left(9x + 5\right) - 4\left(9x + 5\right) = \left(9x + 5\right)\left(7x - 4\right)$$

Occasionally you'll run into a problem that is almost *too easy*. Don't worry, the method still works.

Example: $x^2 + 3x + x + 3$

Work:

$$x^2 + 3x + x + 3 = \left(x^2 + 3x\right) + \left(x + 3\right)$$
$$= x\left(x + 3\right) + 1\left(x + 3\right) = \left(x + 3\right)\left(x + 1\right)$$

For this example there's nothing more complicated than a 1 to factor out of our second pair of terms. That's okay. It'll happen. Just go with it.

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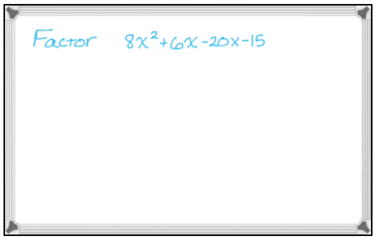

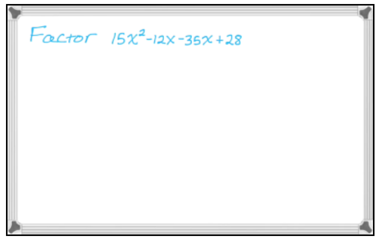

Now go back and work through the other examples. Click here to do that.

Now Do This

1. $x^2+5x+2x+10$ (Sol.)
2. $x^2-3x+4x-12$ (Sol.)
3. $x^2-6x-2x+12$ (Sol.)
4. $6x^2+3x-4x-2$ (Sol.)
5. $15x^2+35x-9x-21$ (Sol.)
6. $24x^2+9x-40x-15$ (Sol.)

Factoring Trinomials

Okay, well, if you've followed along with factoring out a GCF and you've mastered the art of factoring by grouping, you're ready to factor a trinomial. Factoring trinomials starts with a weird process of finding a pair of numbers with a specific product and sum, then rewriting the trinomial, then doing our factoring by grouping thing. So we should be pretty good at this.

The first thing we need to do is to get good at finding our numbers. The numbers that we need meet two criteria: they add up to the coefficient of x in the trinomial and they have a product—they multiply to—the same thing as the product of the coefficient of x^2 and the constant of the trinomial you're factoring.

Say we have $6x^2+7x+2$. We need numbers that sum—that means they add up—to 7, the coefficient of x, and they multiply to 12, which is the product of 6, the coefficient of x^2, and 2, the constant term.

I usually find these numbers one of two ways—either by being really organized about it or by guessing and getting lucky. If I'm being organized I start running through all of the factor pairs (two numbers that multiply to a particular number) and finding the sums. I stop when I hit the factor pair I need.

Factor Pair	1, 12	2, 6	3, 4
Sum	13	8	7

I found it with the factor pair 3 and 4. They multiply to the 12 that I need and they sum to the 7 that I need.

Now the magic happens. We take the middle term of our trinomial, the $7x$, and we rewrite it as a sum using our factor pair as the new coefficients. So we replace $7x$ with $3x+4x$. Now our problem looks familiar! We're trying to factor $6x^2+3x+4x+2$. This will fall to our factor by grouping method from before.

$$6x^2+3x+4x+2=\left(6x^2+3x\right)+\left(4x+2\right)$$
$$=3x\left(2x+1\right)+2\left(2x+1\right)=\left(2x+1\right)\left(3x+2\right)$$

So we're done! This takes some practice, but really it just comes down to finding the right factor pair that will give us the sum and product we need.

Let's do a few more examples.

If you'd rather watch me do some videos first, click here.

Example: Factor $12x^2 + 29x + 15$

Work: We need two numbers that sum to 29 and multiply to $(15)(12) = 180$. Let's be organized about it.

Factor Pair	Sum
1, 180	181
2, 90	92
3, 60	63
4, 45	49
5, 36	41
6,30	36
9,20	29

This was a rough one because we had to go through so many pairs before we found our target sum—but we found it! Now we rewrite and do our group factoring.

$$12x^2 + 29x + 15 = 12x^2 + 20x + 9x + 15$$

$$= \left(12x^2 + 20x\right) + \left(9x + 15\right) = 4x\left(3x + 5\right) + 3\left(3x + 5\right)$$

$$= \boxed{\left(3x + 5\right)\left(4x + 3\right)}$$

Now, it's weird, but always remember that it doesn't matter in which order you write the two middle terms. The method will work either way.

Example: Factor $21x^2 + 37x + 12$

Work: We need two numbers that sum to 37 and multiply to $(21)(12) = 252$.

Factor Pair	Sum
1, 252	253
2, 126	128
3, 84	87
4, 63	67
6, 42	48
7, 36	43
9, 28	37

This was another rough one but we still got it. Now we rewrite and do our group factoring.

$$21x^2 + 37x + 12 = 21x^2 + 9x + 28x + 12$$

$$= \left(21x^2 + 9x\right) + \left(28x + 12\right) = 3x\left(7x + 3\right) + 4\left(7x + 3\right)$$

$$= \boxed{\left(7x + 3\right)\left(3x + 4\right)}$$

Okay, I've been taking it easy on you because all of the coefficients have been positive so far. Now let's do some examples where that's not the case. We need only to keep in mind three things that you've known for ages.

- To get a positive product we need either two positive numbers or two negative numbers—they must have the same sign.
- To get a positive sum we need either two positive numbers or—if the numbers have opposite signs—the "bigger" number to be positive.
- To get a negative sum we need either two negative numbers or—if the numbers have opposite signs—the "bigger" number to be positive.

Example: Factor $6x^2-11x-10$

Work: We need two numbers that sum to -11 and multiply to $(6)(-10)=-60$. Let's be organized about it. If they multiply to a negative, then we know that one number is negative and the other is positive. Since they sum to a negative, we know that the "bigger" number is negative. Let's think about that as we look for our factor pair.

Factor Pair	Sum
1, -60	-59
2, -30	-28
3, -27	-24
4, -15	-11

Found it! Now we rewrite and do our group factoring.

$$6x^2-11x-10=6x^2+4x-15x-10$$
$$=(6x^2+4x)+(-15x-10)=2x(3x+2)-5(3x+2)$$
$$=\boxed{(3x+2)(2x-5)}$$

Example: Factor $10x^2-31x+15$

Work: We need two numbers that sum to -31 and multiply to $(10)(15)=150$. Let's be organized about it. If they multiply to a positive, then we know that they have the same sign. Since they sum to a negative, we know that they must both be negative.

Factor Pair	Sum
-1, -150	-152
-2, -75	-77
-3, -50	-53
-5, -30	-35
-6, -25	-31

Found it! Now we rewrite and do our group factoring.

$$10x^2-31x+15=10x^2-6x-25x+15$$
$$=(10x^2-6x)+(-25x+15)=2x(5x-3)-5(5x-3)$$
$$=(5x-3)(2x-5)$$

A lot of things you'll have to factor are actually much easier than the examples we've been doing. Let's do some of them.

Example: Factor x^2+4x+3

Work: We need two numbers that sum to 4 and multiply to $(1)(3)=3$.

Factor Pair	Sum
1, 3	4

Yeah…so, doing our thing…

$$x^2+4x+3=x^2+3x+x+3$$
$$=(x^2+3x)+1(x+3)=\boxed{(x+3)(x+1)}$$

Example: Factor x^2-5x+6

Work: We need two numbers that sum to -5 and multiply to $(1)(6)=6$. If they multiply to a positive, they have the same sign. If they sum to a negative, they must both be negative.

Factor Pair	Sum
-1, -6	-7
-2, -3	-5

Let's do it.

$$x^2-5x+6=x^2-2x-3x+6$$
$$=(x^2-2x)+(-3x+6)=x(x-2)-3(x-2)$$
$$=\boxed{(x-2)(x-3)}$$

Example: Factor x^2-25

Work: We need two numbers that sum to 0, which is the coefficient of *x* if there's no *x* to be found, and multiply to $(1)(-25)=-25$. If they multiply to a negative, they must have opposite signs. If they sum to 0, they must actually be the same number, just with opposite signs.

Factor Pair	Sum
-5, 5	0

Let's do it.

$$x^2-25=x^2-5x+5x-25$$
$$=(x^2-5x)+(5x-25)=x(x-5)+5(x-5)$$
$$=\boxed{(x-5)(x+5)}$$

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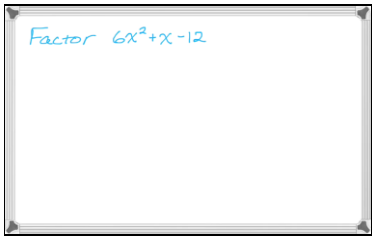

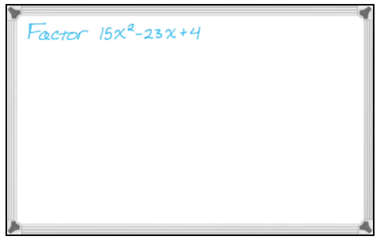

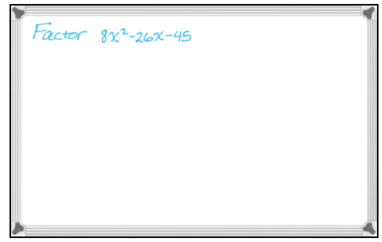

Now go back and work through the other examples! Click here to do that.

Now Do This

Factor each of the following.

1. $x^2+13x+40$ (Sol.)
2. x^2+x-20 (Sol.)
3. $x^2-13x+36$ (Sol.)
4. $8x^2-42x+49$ (Sol.)
5. $21x^2-25x-4$ (Sol.)
6. $24x^2+53x+28$ (Sol.)
7. $20x^2-21x-27$ (Sol.)
8. $6x^2+29x+30$ (Sol.)
9. $63x^2-62x+15$ (Sol.)

And that's the end of that. If you can do everything in the last couple of pages you should be in good shape going forward with factoring. It's a very, very important topic and one that will just utterly destroy you if you avoid learning to do it well. If still have questions, find your teacher! If you're not sure what to ask, just show them the problems you're having trouble understanding. You'll benefit for a long, long time from getting good at factoring.

Can You Distribute? Solutions

1. $7x\left(5x^2+3x+4\right)=7x\left(5x^2\right)+7x\left(3x\right)+7x\left(4\right)=35x^3+21x^2+28x$ (Return)

2. $5x^2\left(3x+2\right)=5x^2\left(3x\right)+5x^2\left(2\right)=15x^3+10x^2$ (Return)

3. $5x\left(x^2-11x+8\right)=5x\left(x^2\right)+5x\left(-11x\right)+5x\left(8\right)=5x^3-55x^2+40x$ (Return)

4.
$\left(7x+3\right)\left(2x-5\right)=\left(7x+3\right)\left(2x\right)+\left(7x+3\right)\left(-5\right)$
$=7x\left(2x\right)+3\left(2x\right)+7x\left(-5\right)+3\left(-5\right)=14x^2+6x-35x-15$
$=14x^2-29x-15$
(Return)

5.
$\left(5x-3\right)\left(3x-2\right)=\left(5x-3\right)\left(3x\right)+\left(5x-3\right)\left(-2\right)$
$=5x\left(3x\right)-3\left(3x\right)+5x\left(-2\right)-3\left(-2\right)=15x^2-9x-10x+6$
$=15x^2-19x+6$
(Return)

6.
$\left(4x+9\right)\left(2x-11\right)=\left(4x+9\right)\left(2x\right)+\left(4x+9\right)\left(-11\right)$
$=4x\left(2x\right)+9\left(2x\right)+4x\left(-11\right)+9\left(-11\right)=8x^2+18x-44x-99$
$=8x^2-26x-99$
(Return)

Greatest Common Factoring Solutions

1. $36x-27=9(4x-3)$ (Return)

2. $5x^2+35x-45=5(x^2+7x-9)$ (Return)

3. $18x^3-3x^2+6x=3x(6x^2-x+2)$ (Return)

4. $4x^4+12x^3-16x^2+80x=4x(x^3+3x^2-4x+20)$ (Return)

5. $9x^3y^2-6x^2y^3=3x^2y^2(3x-2y)$ (Return)

6. $12x^4y^3+24x^2y^4+60xy^4=12xy^3(x^3+2xy+5y)$ (Return)

Factoring by Grouping Solutions

1.
$$x^2+5x+2x+10=(x^2+5x)+(2x+10)$$
$$=x(x+5)+2(x+5)=(x+5)(x+2)$$
(Return)

2.
$$x^2-3x+4x-12=(x^2-3x)+(4x-12)$$
$$=x(x-3)+4(x-3)=(x-3)(x+4)$$
(Return)

3.
$$x^2-6x-2x+12=(x^2-6x)+(-2x+12)$$
$$=x(x-6)-2(x-6)=(x-6)(x-2)$$
(Return)

4.
$$6x^2+3x-4x-2=(6x^2+3x)+(-4x-2)$$
$$=3x(2x+1)-2(2x+1)=(2x+1)(3x-2)$$
(Return)

5.

$15x^2 + 35x - 9x - 21 = (15x^2 + 35x) + (-9x - 21)$

$= 5x(3x + 7) - 3(3x + 7) = (3x + 7)(5x - 3)$

(Return)

6.

$24x^2 + 9x - 40x - 15 = (24x^2 + 9x) + (-40x - 15)$

$= 3x(8x + 3) - 5(8x + 3) = (8x + 3)(3x - 5)$

(Return)

Factoring Trinomials Solutions

1. Factor $x^2 + 13x + 40$

We need a sum of 13 and a product of $(1)(40) = 40$.

Factor Pair	Sum
1, 40	41
2, 20	22
4, 10	14
5, 8	13

Now we rewrite and do our group factoring.

$x^2 + 13x + 40 = x^2 + 5x + 8x + 40$

$= (x^2 + 5x) + (8x + 40) = x(x + 5) + 8(x + 5)$

$= \boxed{(x + 5)(x + 8)}$

(Return)

2. Factor $x^2 + x - 20$

We need a sum of 1 and a product of $(1)(-20) = -20$. Negative product means opposite signs. Positive sum means “bigger” number is positive.

Factor Pair	Sum
20, -1	-19
10, -2	8
5, -4	1

Now we rewrite and do our group factoring.

$x^2 + x - 20 = x^2 + 5x - 4x - 20$

$= (x^2 + 5x) + (-4x - 20) = x(x + 5) - 4(x + 5)$

$= \boxed{(x + 5)(x - 4)}$

(Return)

3. Factor $x^2 - 13x + 36$
We need a sum of -13 and a product of $(1)(36) = 36$. Positive product means same sign. Negative sum means they're both negative.

Factor Pair	Sum
-1, -36	-37
-2, -18	-20
-3, -12	-15
-4, -9	-13

Now we rewrite and do our group factoring.

$x^2 - 13x + 36 = x^2 - 4x - 9x + 36$

$= (x^2 - 4x) + (-9x + 36) = x(x-4) - 9(x-4)$

$= \boxed{(x-4)(x-9)}$

(Return)

4. Factor $8x^2 - 42x + 49$
We need a sum of -42 and a product of $(8)(49) = 392$. Positive product means same sign. Negative sum means both negative.

Factor Pair	Sum
-1, -392	-393
-2, -196	-198
-4, -98	-102
-7, -56	-63
-8, -49	-57
-14, -28	-42

Now we rewrite and do our group factoring.

$8x^2 - 42x + 49 = 8x^2 - 14x - 28x + 49$

$= (8x^2 - 14x) + (-28x + 49) = 2x(4x-7) - 7(4x-7)$

$= \boxed{(4x-7)(2x-7)}$

(Return)

5. Factor $21x^2 - 25x - 4$
We need a sum of -25 and a product of $(21)(-4) = -84$. Negative product means opposite signs. Negative sum means "bigger" number is negative.

Factor Pair	Sum
1, -84	-83
2, -42	-40
3, -28	-25

Now we rewrite and do our group factoring.

$21x^2 - 25x - 4 = 21x^2 + 3x - 28x - 4$

$= (21x^2 + 3x) + (-28x - 4) = 3x(7x+1) - 4(7x+1)$

$= \boxed{(7x+1)(3x-4)}$

(Return)

6. Factor $24x^2+53x+28$

We need a sum of 53 and a product of $(28)(24)=672$. Positive product means same sign. Positive sum means both are positive. Also, I'm going to skip a bunch that couldn't possibly work.

Factor Pair	Sum
8, 84	92
12, 56	68
14, 48	62
16, 42	58
21, 32	53

Now we rewrite and do our group factoring.

$24x^2+53x+28=24x^2+21x+32x+28$

$=(24x^2+21x)+(32x+28)=3x(8x+7)+4(8x+7)$

$=\boxed{(8x+7)(3x+4)}$

(Return)

7. Factor $20x^2-21x-27$

We need a sum of -21 and a product of $(20)(-27)=-540$. Negative product means opposite signs. Negative sum means "bigger" number is negative. Also, I'm going to skip a bunch that couldn't possibly work.

Factor Pair	Sum
9, -60	-51
10, -54	-44
12, -45	-33
15, -36	-21

Now we rewrite and do our group factoring.

$20x^2-21x-27=20x^2+15x-36x-27$

$=(20x^2+15x)+(-36x-27)=5x(4x+3)-9(4x+3)$

$=\boxed{(4x+3)(5x-9)}$

(Return)

8. Factor $6x^2+29x+30$

We need a sum of 29 and a product of $(6)(30)=180$. Positive product means same sign. Positive sum means both are positive. Also, I'm going to skip a bunch that couldn't possibly work.

Factor Pair	Sum
4, 45	49
5, 36	41
6, 30	36
9, 20	29

Now we rewrite and do our group factoring.

$6x^2+29x+30=6x^2+9x+20x+30$

$=(6x^2+9x)+(20x+30)=3x(2x+3)+10(2x+3)$

$=\boxed{(2x+3)(3x+10)}$

(Return)

9. Factor $63x^2 - 62x + 15$

We need a sum of -62 and a product of $(63)(15) = 945$. Positive product means same sign. Negative sum means both negative. Also, I'm going to skip a bunch that couldn't possibly work.

Factor Pair	Sum
-15, -63	-78
-21, -45	-66
-27, -35	-62

Now we rewrite and do our group factoring.

$63x^2 - 62x + 15 = 63x^2 - 27x - 35x + 15$

$= (63x^2 - 27x) + (-35x + 15) = 9x(7x - 3) - 5(7x - 3)$

$= \boxed{(7x - 3)(9x - 5)}$

(Return)