# POLYNOMIALS: INTRODUCTION

## MONOMIALS:

<table>
<thead>
<tr>
<th>EXAMPLES:</th>
<th>NON-EXAMPLES:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-4)</td>
<td>(2^x)</td>
</tr>
<tr>
<td>A number</td>
<td>Variable as an exponent</td>
</tr>
<tr>
<td>(y)</td>
<td>(x^2 + 3)</td>
</tr>
<tr>
<td>A variable</td>
<td>A sum</td>
</tr>
<tr>
<td>(a^2)</td>
<td>(5a^{-2})</td>
</tr>
<tr>
<td>The product of variables</td>
<td>Negative exponent</td>
</tr>
<tr>
<td>(\frac{1}{2}x^2y)</td>
<td>(\frac{3}{x})</td>
</tr>
<tr>
<td>The product of numbers and variables</td>
<td>A quotient</td>
</tr>
</tbody>
</table>

**Examples:** Determine if each expression is a monomial.

1. \(-4xy\)  
2. \(a^2 - 8\)  
3. \(\frac{x}{5}\)  
4. \(7z^{-1}\)  
5. \(b^7\)

### POLYNOMIAL:

A polynomial is a ________________ or the ________________ of different monomials.

**Determine which expressions are polynomials:**

6. \(2q\)  
7. \(d + \frac{c}{d}\)  
8. \(p + q\)  
9. \(\frac{ab - a}{4}\)  
10. \(x^2 + 4x - 8\)  
11. \(7y^3 - 5y^{-2} + 4y\)

### SPECIFIC TYPES OF POLYNOMIALS

<table>
<thead>
<tr>
<th>BINOMIAL:</th>
<th>TRINOMIAL:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples:</td>
<td>Examples:</td>
</tr>
</tbody>
</table>

**Examples #12 - 19:** Determine if each expression is a monomial, binomial, trinomial, or not a polynomial.

12. \(2m - 7\)  
13. \(x^2 + 3x - 4 - 5\)  
14. \(\frac{5}{2x} - 3\)  
15. \(3y^2 - 6 + 7y\)
16. $3x + 8x - 5x^2$  
17. $8x^3y^2z$  
18. $2a^2 + 3ab - 5ba$  
19. $9r + 11 - 5r^2$

**DEGREE**: Based on the exponents of the variables.
- **The degree of a MONOMIAL:**

- **The degree of a POLYNOMIAL:**

**Examples**: Find the degree of each polynomial.

20. $5mn^2$  
21. $9x^3yz^6$  
22. $5a^2 + 3$  
23. $-4x^2y^2 + 3x^2 + 12$  
24. $3x^2 - 7x$  
25. $8m^3 - 2m^2n^2 - 11$

**REORDERING TERMS OF A POLYNOMIAL BASED ON DEGREE**:  
**MOVE TERMS AND KEEP THE SIGN WITH THE TERM**

**Example**: Arrange the polynomials in **descending order** according to the powers of the $x$.

a) $6x^2 + 5 - 8x - 2x^3$  
   d) $3a^3x^2 - a^4 + 4ax^5 + 9a^2x$  

b) $7x^2 - 11x^4 + 8 - 2x^5$  
   e) $15x^5 - 2x^2y^2 - 7yx^4 + x^3y$

c) $25x^6 - 3x^2 + 7x^5 + 15x^8$
POLYNOMIALS: ADDITION AND SUBTRACTION

WARM UP ACTIVITY: Simplify the following
1) $3x - 2y + 4y - 6x$     3) $4z + 2t + 3z - t$     5) $8a + 6b + 6a + 2b$

2) $3x - 12y - 2x^2 + 6y$     4) $5a + 3b - 2c - 8a$

ADDING AND SUBTRACTING POLYNOMIALS:
- When adding and subtracting polynomials, you **combine like terms**.
- Be careful of parentheses and positive or negative signs with the operations.

Exp 1: $(3x^2 - 4x + 8) + (2x - 7x^2 - 5)$
Exp 2: $(3n^2 + 13n^3 + 5n) - (7n + 4n^3)$
Exp 3: $(2b^2 + 8ab^3 + 4b) - (9b - 5ab^3)$
Exp 4: $(6y^2 + 8y^4 - 5y) - (9y^4 - 7y + 2y^2)$
Exp 5: $(7y^2 + 2y - 3) + (2 - 4y + 5y^2)$
Exp 6: $(3x^2 + 5x + 2) - (4 - 2x) + (5x^2 + 7)$

PRACTICE PROBLEMS: Simplify each expression
1. $x^2 + 2x - 3 + 2x^2 - 7x + 9$     2. $(3x + 5) + (2x - 3)$
3. \((-2x + 3) + (4x - 3)\) 
4. \((2x^2 + 2x - 4) + (x^2 + 3x + 7)\)

5. \((3a^2 + a - 4) + (a^2 - 2a - 1)\) 
6. \((t^2 - 1) + (2t + 3)\)

7. \((2x^2 + 3) + (x^2 - 2x - 1)\) 
8. \((2x^2 + 5xy + 3y^2) + (8x^2 - 7y^2)\)

9. \((z^2 + 2z - 5) + (3z^2 - z + 4)\) 
10. \((4m - 3n) + (2n + 5m)\)

11. \((6x + 5) - (3x + 1)\) 
12. \((2a^2 - 3a) + (5b - b^2) + (2a - 8b)\)

13. \((3z^2 + 5) - (-4z + 2z^2) - (z - 3)\) 
14. \((3x - 2) - (5x - 4) + (19 + 2x)\)

15. \((10x^2 + 8x) - (6 + 3x^2) + (2x - 9)\) 
16. \((6m^2 + 7) - (-2m^2) + (2m - 3)\)
Find the PERIMETER of the shape.

Equation: Perimeter = Sum of all the sides

4x - 8
11 + y

3a - b

2y - 3x - 3
12 + 5x + 7y

3b - 4a + 5
6 + 2a

7 + 3x
5x^2 - 2

3x^2 + 5x + 7
6x - 3

3ab + 4a^2
2a^2

3x^2 - 4x
7 - 2x

3x^2 - 4x
7 - 2x

5x^2 + 7x + 3
Find the sum or difference:

1) \((x^3 - 7x + 4x^2 - 2) - (2x^2 - 9x + 4)\)

2) \((3a + 2b - 7c) + (6b - 4a + 9c)\)

3) \((5y^2 - 2xy + 6x^2 - 3x + 7y - 9) + (3x^2 - 4x + 5) - (5y^2 - 3y + 6)\)

Word Problems:

1) Bob mowed \((2x^2 + 5x - 3)\) yards on Monday, \((4x - 7)\) yards on Tuesday, and \((3x^2 + 10)\) yards on Wednesday.
   a. How many yards did he mow in the three days?

   b. If Bob mowed \(14x^2 + 12x - 3\) yards total for the entire week, how many yards did he mow during the rest of the week?

2) Molly has \((4x + 10)\) dollars and Ron has \((-5x + 20)\) dollars.
   a. How much money do they have altogether?

   b. How much more money does Molly have than Ron?

3) Ross has \((8x - 5)\) tickets for Chuck E Cheese. He is going to play today and wants to buy a prize that is \((15x + 1)\) tickets. How many tickets must he win to have enough tickets to buy the prize?
4) The measure of the perimeter of a triangle is $37s + 42$. It is known that two of the sides of the triangle have measures of $14s + 16$ and $10s + 20$. Find the length of the third side.

5) A triangle has a perimeter of $10a + 3b + 12$ and has sides of length $3a + 8$ and $5a + b$, what is the length of the third side?

6) For a rectangle with length of $3x + 4$ and perimeter of $10x + 18$, what is the width of the rectangle?

7) A rectangle has a perimeter of $12y^2 - 2y + 18$ and has a width of $4y^2 - y + 6$. What is the length of the rectangle?
POLYNOMIALS: Multiplication of Monomial and Polynomial

DISTRIBUTIVE PROPERTY REVIEW

1) \(-4(2 - 6x)\)  
2) \(3(5p + q - 3r)\)  
3) \(-2(-x - 7y)\)

SIMPLIFYING PRACTICE PROBLEMS:

1) \((4x + 7x)^3\)  
2) \(12z - 5z + 9z^2\)  
3) \(-7(-6m + 11m)\)

4) \(4(11 - 3x)\)  
5) \(-5(5a - 3b - 6)\)  
6) \(-2(x^2 - 8x + 3x^3 - 6)\)

7) \(9x - 4(6 - 3x)\)  
8) \(5(3b - 2a) - 7b\)  
9) \(12 + 3(7x + 2)\)

10) \(6(4y + 3z) - 11z\)  
11) \(5 + 2(4m - 7n) + 9n\)  
12) \(12 - 7(3 - 5r) + 8r\)
13) $19x + 1(2 + 4x) - 18$

14) $2(2x + 6) + 3(5x - 7)$

15) $6(4a - 2b) - 2(9b - 7a)$

16) $5(3x + 2y) - 4(7y + 8z)$

**LAWS of EXPONENTS REVIEW:**
Multiply Coefficients and Add Exponents of Same Variable

1) $(3x^2)(7x^3)$

2) $8m^5 \cdot m$

3) $t^3 \cdot 6t^7$

4) $(4y^4)(-9y^2)$

5) $3r^5 \cdot 2r^2 \cdot 7r^6$

6) $(-2p^3r)(11r^4p^6)$

7) $(6y^3x)(5y^3)$

8) $7c^5a^3b \cdot 8a^2b^4c$

9) $(-3t^3u^2)(-4u^3t)$

**Using Law of Exponents and Distributive Property:**

1) $4x(2x + 6)$

2) $9y^2(5y - 3)$

3) $-6a(3a^2 - 7a - 11)$

4) $3z^3(12z + 4z^2 - 1)$

5) $2pq(3p^2 + 6pq + 7q^2)$

6) $-5xy^3(-3x^3 + 7y - 2xy)$
MULTIPLYING A POLYNOMIAL BY A MONOMIAL:
USE THE DISTRIBUTIVE PROPERTY with VARIABLE TERMS
Keep track of Coefficients and Exponents of Variables

Exp 1: $y(y + 5)$  
Exp 2: $-2n(7 - 5n^2)$

Exp 3: $-7m (3m^2 + 4m + 5)$  
Exp 4: $2ab (3a^2 - 2ab + 6b^2)$

Exp 5: $3a^3 (2a^2 - 5a + 8)$  
Exp 6: $-3x^3y (5yx + 6y^2)$

BOX METHOD: $6y^2 (4y^2 - 9y - 7) =

Practice. Simplify each example
1. $7(2x + 5)$  
2. $4x(3x^2 - 7)$  
3. $-5a(6 - 3a^2)$

4. $2m^2(5m^2 - 7m + 8)$  
5. $3r(-2r^2 + 6r - 5)$  
6. $6x^3y(-x + 7y - 3xy)$
7. $2y^2(7y + 3x) - 5y^3$
8. $11y(y - 3) + 13y$
9. $3(x^3 + 4x^2) + 2x(x - 7)$

10. $4(3d^2 + 5d) - d(d^2 - 7d + 12)$
11. $3(2t^2 - 4t - 15) + 6t(5t + 2)$

**SPECIAL PROBLEMS:** Find the area of the shaded region in the simplest form.

**(BIG SHAPE) – (LITTLE SHAPE “HOLE”) = SHADED REGION**

**EXAMPLES:**
1) A square of side length 8 has a triangle of base 4 and height 3 cut out of it.

2) A rectangle with width of 7 and length of 9 has a square of side length 5 cut out of it.

3) $3t - 8$
4) $5x - 2$
5) $3 - t$

$6y$ $6y$

$11y$

$3x$ $3x$

$11y$

$3t$
**POLYNOMIALS: FOIL BOX METHOD Part 1**

**FOIL Box Method:** The box method does the exact same multiplications as our standard FOIL method, but gives it in a graphic organizer.

- Be careful of positive and negatives.
- Combine like terms of boxes to finish.

<table>
<thead>
<tr>
<th>Binomial #1</th>
<th>Binomial #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>F first terms</td>
<td>O outer terms</td>
</tr>
<tr>
<td>I inner terms</td>
<td>L last terms</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exp 1: ((x + 2) (x + 1))</th>
<th>Exp 2: ((y + 3) (y - 4))</th>
<th>Exp 3: ((a - 5) (a - 7))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exp 4: ((3x + 2) (x + 4))</th>
<th>Exp 5: ((5b + 9) (b - 4))</th>
<th>Exp 6: ((2n - 7) (3n + 3))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exp 7: ((2x - 5) (2x - 5))</th>
<th>Exp 8: ((8r^2 - 2r) (5r + 4))</th>
<th>Exp 9: ((2x + 5y) (7y - 3x))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Practice Problems: Multiply the following binomials.

1. \((x - 3)(x - 2)\) 
2. \((2x + 1)(x + 1)\) 
3. \((y + 4)(y - 2)\) 

4. \((x - 7)(x - 3)\) 
5. \((y - 4)(2y + 3)\) 
6. \((4h - 3)(3h + 2)\) 

7. \((m - 3)(m + 1)\) 
8. \((2a - 3)(a - 2)\) 
9. \((3x + 1)(x + 2)\) 

10. \((2x - 3)(2x + 2)\) 
11. \((3a - b)(2a + 4b)\) 
12. \((2x + y)(3x - 2y)\) 

13. \((x^2 - 4)(x + 3)\) 
14. \((x^2 + 6x)(x - 1)\) 
15. \((a^2 + 2)(a^5 + 1)\)
### WARM UP: Simplify each expression by FOIL

1) \((3b - 5)(b - 4)\)  
2) \((y + 7)(y + 6)\)  
3) \((2n + 9)(n - 8)\)

### BINOMIAL TIMES TRINOMIAL: One More Column for 3rd term in trinomial

<table>
<thead>
<tr>
<th>Example 1: ((a + 3) (a^2 + 7a + 6))</th>
<th>Example 2: ((4x + 9) (2x^2 - 5x + 3))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Example 3: ((y - 5) (4y^2 - 3y + 2))</th>
<th>Example 4: ((2b + 1) (b^2 - 5b + 4))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Example 5: ((x - 6) (x^2 - 7x - 8))</th>
<th>Example 6: ((3b^2 - 4b) (2b^2 - b + 7))</th>
</tr>
</thead>
</table>